

**GROUPS AND REPRESENTATIONS I: PROBLEM SET 3**  
**Due Wednesday, October 24**

**Problem 1:** Using Cartan's criterion for semisimplicity, show that any derivation  $D$  of a semi-simple Lie algebra  $L$  is of the form

$$D = ad(X)$$

for some  $X \in L$ .

**Problem 2:** Show that the decomposition of a representation of a semisimple Lie algebra into irreducibles is unique.

**Problem 3:** Show that the Casimir operator  $C_\pi$  of  $\mathfrak{sl}(2, \mathbf{C})$  defined in class commutes with all  $\pi(X)$  for  $X \in \mathfrak{sl}(2, \mathbf{C})$ .

**Problem 4:** Knapp Chapter I, problem 17 (Note, this was essentially worked out in class)

**Problem 5:** Knapp Chapter I, problems 31-35