Groups and Representations I: Problem Set 3
Due Wednesday, October 24

Problem 1: Using Cartan’s criterion for semisimplicity, show that any derivation $D$ of a semi-simple Lie algebra $L$ is of the form

$$D = ad(X)$$

for some $X \in L$.

Problem 2: Show that the decomposition of a representation of a semisimple Lie algebra into irreducibles is unique.

Problem 3: Show that the Casimir operator $C_\pi$ of $\mathfrak{sl}(2, \mathbb{C})$ defined in class commutes with all $\pi(X)$ for $X \in \mathfrak{sl}(2, \mathbb{C})$.

Problem 4: Knapp Chapter I, problem 17 (Note, this was essentially worked out in class)

Problem 5: Knapp Chapter I, problems 31-35