Lie Groups and Representations, Spring 2016
Problem Set 8

Due Monday, April 4

Problem 1: Use the Chevalley-Eilenberg complex to show that $H^3(\mathfrak{sl}(2, \mathbf{C}) = \mathbf{C}$ and give a representative cocycle.

Problem 2: Prove that the Lie algebra cohomology $H^2(\mathfrak{g}, \mathbf{C})$ classifies the central extensions

$$0 \to \mathbf{C} \to \mathfrak{g}' \to \mathfrak{g} \to 0$$

(i.e. extensions of this form where the subalgebra $\mathbf{C} \subset \mathfrak{g}'$ commutes with everything in $\mathfrak{g}'$.)

Problem 3: Use the Borel-Weil-Bott theorem to find the cohomology groups $H^*(G/B, \mathcal{O}(L_\lambda)$ when $G = SL(3, \mathbf{C})$ for $\lambda = n_1\omega_1 + n_2\omega_2$, where n_1, n_2 are integers and ω_1, ω_2 are the fundamental weights.

Problem 4: Calculate the Lie algebra cohomology $H^*(\mathfrak{g}, \mathbf{C})$ for $\mathfrak{g}$ the Lie algebra of strictly upper triangular 3 by 3 complex matrices. (Hint: this is $\mathfrak{n}_+$ for $\mathfrak{sl}(3)$.)