Problem 1: Prove Frobenius reciprocity

\[ \text{Hom}_G(\text{Ind}_H^G(W)) = \text{Hom}_H(V_H, W) \]

for \( H \) a closed subgroup of a Lie group \( G \), \( W \) a representation of \( H \), \( V \) a representation of \( G \).

Problem 2: Adapt the Borel-Weil (holomorphic induction) constructions given in chapters 11 and 14 of Graeme Segal’s *Lectures on Lie groups and Lie algebras* to the case of \( SL(n, \mathbb{C}) \) (i.e. give a detailed construction of the irreducible representations corresponding to \( k \omega_1 \) and \( \omega_2, \cdots, \omega_{n-1} \) as holomorphic functions on the group).