Lie Groups and Representations, Fall 2013
Problem Set 3

Due Monday, October 21

Problem 1: Kirillov Problem 4.5

Problem 2: (a) Show that

$$\pi : t \to \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

gives a representation of the group $\mathbf{R}$ on $\mathbf{C}^2$.
(b) Find all subrepresentations.
(c) Show this this representation is not unitary, that is is reducible, but not completely reducible.

Problem 3: Kirillov Problem 4.7

Problem 4: Kirillov Problem 4.10

Problem 5: Prove that the Frobenius-Schur indicator

$$\frac{1}{|G|}\sum_{g\in G}\chi_V(g^2)$$

for a complex irreducible representation V takes three possible values: $-1, 0, 1$. For real irreducible representations, what are the possibilities for

$$Hom_G(V,V)$$

? Which ones does one get when restricting the three possible sorts of complex irreducible representations given above?
(Apologies for the initial confused version of this problem!!)
Note: this is not covered in Kirillov, and you should try consulting some other sources to learn more about this. One place that has a good discussion is the textbook of Fulton and Harris, in section 3.5.