Problem 1: Kirillov Problem 4.5

Problem 2: (a) Show that
\[ \pi : t \rightarrow \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \]
gives a representation of the group \( \mathbb{R} \) on \( \mathbb{C}^2 \).
(b) Find all subrepresentations.
(c) Show this representation is not unitary, that is it is reducible, but not completely reducible.

Problem 3: Kirillov Problem 4.7

Problem 4: Kirillov Problem 4.10

Problem 5: Prove that the Frobenius-Schur indicator
\[ \frac{1}{|G|} \sum_{g \in G} \chi_V(g^2) \]
for a complex irreducible representation \( V \) takes three possible values: \(-1, 0, 1\).
For real irreducible representations, what are the possibilities for
\[ \text{Hom}_G(V, V) \]
? Which ones does one get when restricting the three possible sorts of complex irreducible representations given above?
(Apologies for the initial confused version of this problem!!)
Note: this is not covered in Kirillov, and you should try consulting some other sources to learn more about this. One place that has a good discussion is the textbook of Fulton and Harris, in section 3.5.