# Lie Groups and Representations, Fall 2013
## Problem Set 1

**Due Monday, September 23**

**Problem 1:** Prove that the matrix groups $SO(n)$ and $SU(n)$ are compact and connected.

**Problem 2:** Show that $SU(n)/SU(n-1) = S^{2n-1}$ and $SO(n)/SO(n-1) = S^{n-1}$.

**Problem 3:** Prove that the set of right-invariant vector fields forms a Lie algebra under the Lie bracket operation, and show that it is isomorphic to $T_1G$. Define the diffeomorphism

$$\phi : g \in G \to \phi(g) = g^{-1} \in G$$

Show that if $X$ is a left-invariant vector field, then $d\phi(X)$ is a right-invariant vector field, whose value at 1 is the same as that of $-X$. Show that

$$X \to d\phi(X)$$

gives an ismorphism of the Lie algebras of left and right invariant vector fields on $G$.

**Problem 4:** Exercise 2.5 in Kirillov

**Problem 5:** Exercises 2.8, 2.9 and 2.10 in Kirillov

Note: See chapter 6 of `http://www.math.columbia.edu/~woit/QM/qmbook.pdf` for a detailed discussion of this situation, using quaternions, and you can use this instead to solve these problems if you wish.