Groups and Representations, Spring 2012
Problem Set 1
Due Monday, February 6

Problem 1: Prove the Frobenius reciprocity theorem

$$Hom_G(V, Map_H(G, W)) = Hom_H(V, W)$$

by constructing explicitly maps in both directions.

Problem 2: Decompose the space of complex -valued functions on S^2 into orthonormal functions Y_m^l of spherical coordinates as described in class, using Frobenius reciprocity and what you know about $SU(2)$ representations. Find explicit formulas for the Y_m^1 and show how $SU(2)$ and $SO(3)$ act on these functions.

Problem 3: Given two distinct irreducible representations V_1 and V_2 of a compact Lie group G, prove that two functions on G are orthogonal in $L^2(G)$ if one is a matrix element in the representation V_1, the other is a matrix element in the representation V_2.