Practice Final Exam
Calculus I Spring 2007
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1. Compute the following limits (show your work and indicate your reasoning):

   (a)
   $$\lim_{x\to\infty} \frac{\sin x}{x}$$

   (b)
   $$\lim_{x\to\infty} \frac{3x^3+6}{e^x}$$

   (c)
   $$\lim_{x\to 0} x^x$$

   (d)
   $$\lim_{x\to 0^+} (2x)\ln(3x)$$

2. Differentiate the following functions:

   (a) $f(x) = \tan(3x^2+1)$
   (b) $g(x) = 5^{x-2}$
   (c) $h(x) = \arcsin(2x)$
   (d) $k(x) = \ln(\ln(5x^2))$
   (e) $l(x) = \frac{3x^2+1}{2x-5}$
   (f) $m(x) = (x^3+1)^{12}$
   (g) $p(x) = \int_{-x}^{x} \sqrt{1+\sin^2 t}\,dt$

3. Find antiderivatives for the following functions:

   (a) $f(x) = 3x^4 + 8x + 2$
   (b) $g(x) = \sec(2x)\tan(2x)$
   (c) $h(x) = \tan(2x)\sec^2(2x)$
   (d) $k(x) = 10^{2x}$
   (e) $l(x) = \frac{4x+12}{(x^2+6x)^3}$
   (f) $m(x) = \frac{1}{3x+5}$
   (g) $n(x) = \sin(2x) + \cos(2x)$
   (h) $p(x) = x^2 e^{x^3}$

4. Evaluate the following definite integrals:

(a)
$$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$$

(b)
$$\int_0^1 \sqrt{1+3x}\,dx$$

(c)
$$\int_0^3 x\sqrt{9-x^2}\,dx$$

(d)
$$\int_0^4 -\sqrt{4-(x-2)^2}\,dx$$

Hint: Draw a picture!

(e)
$$\int_0^1 \frac{d}{dx}e^{\arctan x}\,dx$$

5. Find the volume of the solid obtained by rotating the graph of $f(x) = \cos x$ $(0 \le x \le \pi/2)$ around the x-axis. *Hint:* $\cos^2 x = 2\cos(x/2) - 1$.

6. Use Newton's Method to give a recursive formula (i.e. express the $(n+1)^{st}$ approximation x_{n+1} as a function of the n^{th} approximation x_n) for approximating a root of $f(x) = x^3$. Show that no matter what inital approximation x_1 is taken, the limit
$$\lim_{n\to\infty} x_n$$
is a root of $f(x)$. *Hint:* Write out x_1, x_2, x_3 in terms of x_1 then give a general (not recursive!) formula for x_n in terms of x_1.

7. State both parts of the Fundamental Theorem of Calculus. Name the mathematicians generally credited with the invention of calculus. In what century did they live?

8. Express
$$\int_0^{\pi/2} \sqrt{\sin x}\,dx$$
as a limit of Riemann sums.

9. Show that $f(x) = e^x + x$ has exactly one root.

10. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis. Sketch the region and a typical shell.

    (a) $x = 2y^2 - y^3$, $x = 0$
    (b) $y = 4x^2$, $2x + 6 = y$