① Second part: Range (cosh) $= [1, \infty)$ so for arccosh x to be defined, we assume $x \in [1, \infty)$. When defining arccosh we (by convention) restrict the domain of cosh to $[0, \infty)$ in order to make it 1-1, so $y = \operatorname{arccosh} x \in [0, \infty)$ hence $\sinh y \geq 0$. Now let $y = \operatorname{arccosh} x$ so that $\cosh y = x$ and differentiate implicitly to get $\frac{dy}{dx} \sinh y = 1$

and use the identity $\cosh^2 y - \sinh^2 y = 1$ to conclude $\sinh y = \sqrt{\cosh^2 y - 1}$ (we take the positive square root because of the above discussion) hence $\sinh y = \sqrt{x^2 - 1}$ so $\frac{d}{dx} \operatorname{arccosh} x = \frac{1}{\sqrt{x^2 - 1}}$.

② You may disregard (c) for now, though we will learn how to do it later.

(a) $\frac{1}{2} x^6 + \frac{7}{3} x^3 + 5x$ (you may add any constant of course)

(b) $-\frac{1}{2} \cos(2x)$

(d) $\frac{1}{3} \tan(3x)$

③ (a) $= \lim_{x \to 0^+} \frac{\ln x}{x^{-2/3}}$

$= \lim_{x \to 0^+} \frac{\left(\frac{1}{x}\right)}{\left(-\frac{2}{3}\right) x^{-5/3}}$ by L'Hospital's rule

$= \lim_{x \to 0^+} \left(-\frac{3}{2}\right) \frac{x^{5/3}}{x} = \lim_{x \to 0^+} \left(-\frac{3}{2}\right) x^{2/3} = 0.$

(b) Repeatedly apply L'H. R. to see

$$= \lim_{x \to \infty} \frac{5 \cdot 4 \cdot 3 \cdot 2}{e^x} = 0.$$

⑥ These are surely in the textbook.

⑦ (a) $f'(x) = -2 \sin x + 2 \sin x \cos x$

$$= 2 \sin x (\cos x - 1)$$

(forget the theta; just write x)

so $f'(x) = 0$ iff ($\sin x = 0$ or $\cos x = 1$)

iff $x = n\pi$ for some integer n.

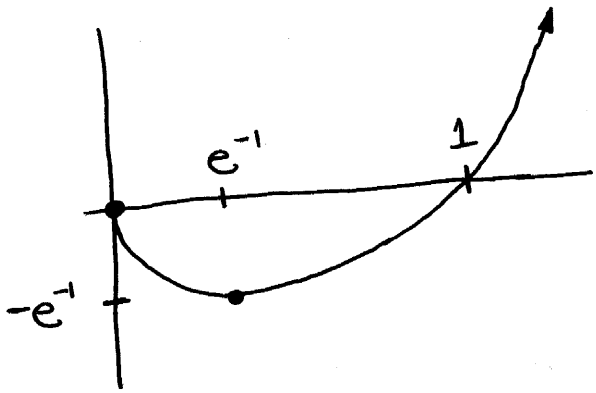
The expression $(\cos x - 1)$ is always non-positive, so the sign of $f'(x)$ is the opposite as the sign of $\sin x$.

It now follows from the First Derivative Test that f has local maxima at even integer multiples of π (in which case $f = 2$) and f has local minima at odd integer multiples of π (in which case $f = -2$).

(b) $h'(x) = 1 + \ln x$ is zero only when $x = e^{-1}$; when $x > e^{-1}$ then $h'(x)$ is positive and when $x \in (0, e^{-1})$, $h'(x)$ is negative (note that the domain of h is $(0, \infty)$) so e^{-1} is a local minimum.

(In some sense, since $\lim_{x \to 0^+} x \ln x = 0$, we could extend h to a continuous function in which case $h(0) = 0$ could be regarded as a local maximum.)

The graph of h looks like:



(c) Hint: Divide into cases: either $x \geq 2$ or $x < 2$.

(9) These are pretty straight-forward. Let's just do (c).

$$h'(x) = (4-2x)e^{4x-x^2}$$

$$h''(x) = ((4-2x)^2 - 2)e^{4x-x^2}$$

$$= (4x^2 - 8x + 14)e^{4x-x^2}$$

Of course, $e^{4x-x^2} > 0$ for all x, so the sign of $h''(x)$ and the ~~zero~~ roots of $h''(x)$ are just those of the quadratic

$$4x^2 - 8x + 14.$$

I leave it to you to finish this.

(10) $x_1 + 2x_2 = 100 \Rightarrow x_1 = 100 - 2x_2$

so just maximize

$$x_1 x_2 = (100 - 2x^2)x_2 \; !$$

This is easy.

(11) Certainly $\lim_{x \to \pm\infty} f(x) = 0$ so $y = 0$ is a horizontal asymptote and $x = 0$ is a vertical asymptote (note that the ~~#~~ numerator is non-zero when $x = 0$!)