

PRACTICE FOR MIDTERM 2
CALCULUS 1 - GILLAM

① Recall that $\sinh x := \frac{e^x - e^{-x}}{2}$, $\cosh x := \frac{e^x + e^{-x}}{2}$,

and that $\tanh x$, $\operatorname{sech} x$, etc. are defined by taking the usual ratios of $\sinh x$ and $\cosh x$.

Show that $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$.

Use implicit differentiation to show that

$$\frac{d}{dx} \operatorname{arccosh} x = \frac{1}{1-x^2}.$$

② Find antiderivatives of the following functions:

(a) $3x^5 + 7x^2 + 5$

(b) $\sin(2x)$

(c) $\frac{1}{1-4x^2}$

(d) $\sec^2(3x)$

③ Calculate the following limits

(a) $\lim_{x \rightarrow 0^+} x^{2/3} \ln x$

(b) $\lim_{x \rightarrow \infty} \frac{x^5}{e^x}$

④ Suppose f is one-to-one, $f(7) = 3$, $f'(7) = 8$.

Find (a) $f^{-1}(3)$

(b) $(f^{-1})'(3)$

⑤ Find the first, second, and third derivatives of the following functions; give a formula for the n^{th} derivative.

(a) $f(x) = 3^x$

(b) $g(x) = \sin(2x)$

⑥ State the Extrema Value Theorem.

State the Mean Value Theorem.

⑦ Find the critical numbers of the following functions. Which are local maxima/minima?

(a) $f(x) = 2 \cos \theta + \sin^2 \theta$

(b) $h(x) = x \ln x$

(c) $g(x) = |3x - 6|$

⑧ Suppose f is a differentiable function on $[0, 10]$ such that $f(0) = 4$ and $f(10) = 19$. Suppose that $f'(x) \leq \frac{3}{2}$ for every x in $[0, 10]$. Give an expression for f on $[0, 10]$ (prove that f is given by this expression).

⑨ Find the intervals of concavity and the inflection points of the following functions.

(a) $f(x) = 2 - 2x - x^3$

(c) $h(x) = e^{4x - x^2}$

(b) $g(x) = \frac{1}{1 - x^2}$

(d) $k(x) = 2 \sin(2x)$

⑩ Find two positive numbers so that the sum of the first number and twice the second number is 100 and the product of the two numbers is as large as possible.

⑪ Draw a graph of $f(x) = \frac{x^2 - 1}{x^3}$. Calculate asymptotes, intervals of increase/decrease, concavity, inflection points, and local maxima/minima.

- ⑫ Suppose $f(x) = ax^3 + bx^2 + cx + d$ has a global maximum at $x = 4$. What can you say about a, b, c, d ?
- ⑬ Find the point on the hyperbola $xy = 8$ closest to $(3, 0)$.
- ⑭ Use Newton's Method to give a recursive formula for approximating a root of $f(x) = x^2 - \cos^2 x$.
- ⑮ A manufacturer has been selling 1000 televisions a week at \$450 each. A market survey indicates that for each \$10 rebate offered to the buyer, the number of televisions sold per week will increase by 100. If the cost of manufacturing x televisions per week is $68,000 + 150x$, then how should the manufacturer set the rebate in order to maximize profit?
- ⑯ Express the following sums as polynomials in n :
- (a) $\sum_{i=1}^n (3i^2 - 2i + 4)$ (b) $\sum_{i=1}^n 2((i+1)^4 - i^4)$
- ⑰ Express the following sums in sigma notation.
- (a) $4 + 6 + 8 + \dots + 2n$
- (b) $2 + 16 + 54 + 128 + \dots + 2n^3$