CALCULUS 3 / SAMPLE MIDTERM 2

70 minutes. No book. No calculator.

(1) Evaluate the integral

$$\int_0^4 (\sqrt{t}\mathbf{i} + te^{-t}\mathbf{j} + \frac{1}{t^2}\mathbf{k})dt$$

(2) The curves $\mathbf{r}_1(t) = \langle t, t^2, t^3\rangle$ and $\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t\rangle$ intersect at the origin. Find their angle of intersection correct to the nearest degree.

(3) Find the curvature of $\mathbf{r}(t) = \langle e^t \cos t, e^t \sin t, t\rangle$ at the point $(1, 0, 0)$.

(4) Paramerize the curve $\mathbf{r}(t) = \langle 3\sin t, 4t, 3\cos t\rangle$ with respect to arc length measured from the point where $t = 0$ in the direction of increasing t.

(5) Find the equations of the osculating circles of the ellipse $9x^2 + 4y^2 = 36$ at the points $(2, 0$ and $(0, 3)$.

(6) Find the vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ at the point $(0, \pi, -2)$ of the curve $\mathbf{r}(t) = \langle 2\sin 3t, t, 2\cos 3t\rangle$. Find the equation of the osculating plane at the same point.

(7) A gun is fired with angle of elevation 30 degree. What is the muzzle speed if the maximum height of the shell is 500?