

CALCULUS 3 / SAMPLE MIDTERM 1

70 minutes. No book. No calculator.

(1) For which value(s) of x are the two vectors $\mathbf{u} = \langle x, 1, x \rangle$, $\mathbf{v} = \langle x, 2, 4 \rangle$ orthogonal ? Parallel ? Of equal length ?

(2) Find parametric equations for the line through $A = (1, 2, 3)$ and $B = (0, 2, 2)$. Find an equation for the plane through $C = (-1, 3, 4)$, $D = (2, 3, 4)$ and $E = (-1, 3, 3)$. Find the intersection between that line and that plane. Find the intersection between that line and the sphere of equation $x^2 + y^2 + z^2 = 1$.

(3) Find the cosine of the angle between a diagonal of a cube (i.e. a line joining opposite vertices through the center of the cube) and one of its edge.

(4) Convert the polar equation $r = 4 \cos \theta$ from polar coordinates to cartesian coordinates and identify the curve it describes.

(5) If $\mathbf{v} \times \mathbf{w} = \langle 1, 2, 2 \rangle$ and $\mathbf{v} \bullet \mathbf{w} = -3$, find the angle between \mathbf{v} and \mathbf{w} .

(6) Let \mathbf{u}, \mathbf{v} be two vectors in the plane. Prove that

$$|\mathbf{u} + \mathbf{v}|^2 + |\mathbf{u} - \mathbf{v}|^2 = 2|\mathbf{u}|^2 + 2|\mathbf{v}|^2$$

(7) Evaluate the following expressions and write your answer in the form $a + bi$.

$$(2 + 3i)(5 + 7i)$$

$$(1 + 2i)^2 + (1 + i)$$

$$\frac{1 + 4i}{3 - 3i}$$

$$|10 - 5i|$$

$$(2/2 - i2/2)^{2006}$$