CALCULUS 3 / SAMPLE FINAL

150 minutes. No book. No calculator.

(1) Find parametric equations for the line through the point $(0, 1, 2)$ that is parallele to the plane $x+y+z = 2$ and perpendicular to the line $x = 1+t, y = 1-t, z = 2t$.

(2) An ellipsoid is created by rotating the ellipse $4x^2 + y^2 = 16$ about the x-axis. Find an equation of the ellipsoid.

(3) A wagon is pulled a distance of $100m$ along a horizontal path by a constant force of $50N$. The handle of the wagon is held at an angle of 30 degree above the horizontal. How much work is done?

(4) Find an equation of a parabola that has curvature 4 at the origin.

(5) The position function of a particle is given by $\mathbf{r}(t) = \langle t^2, 5t, t^2 - 16t\rangle$. When is the speed a minimum?

(6) Find equations of the normal plane and osculating plane of the curve of parametric equation $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$ at the point $(2, 4, 8)$.

(7) Find the following limits if they exist

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2}, \qquad \lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4 + y^4}$$

(8) Find the domain of definition of the following function and show it is differentiable on its domain:

$$f(x, y) := e^x \cos(xy)$$

Then find the linearization of this function at the point $(0, 0)$.

(9) The radius of a right circular cone is increasing at a rate of $1.8in/s$while its height is decreasing at a rate of $2.5in/s$. At what rate is the volume of the cone changing when the radius is is 120 in. and the height is 140 in. ?

(10) Find the dimensions of a rectangular box with largest volume if the sum of the lengths of its 12 edges is a constant c.