CALCULUS 3 / SAMPLE FINAL

150 minutes. No book. No calculator.

(1) Find parametric equations for the line through the point \((0, 1, 2)\) that is parallel to the plane \(x + y + z = 2\) and perpendicular to the line \(x = 1 + t, y = 1 - t, z = 2t\).

(2) An ellipsoid is created by rotating the ellipse \(4x^2 + y^2 = 16\) about the \(x\)-axis. Find an equation of the ellipsoid.

(3) A wagon is pulled a distance of 100\(m\) along a horizontal path by a constant force of 50\(N\). The handle of the wagon is held at an angle of 30 degree above the horizontal. How much work is done?

(4) Find an equation of a parabola that has curvature 4 at the origin.

(5) The position function of a particle is given by \(\mathbf{r}(t) = \langle t^2, 5t, t^2 - 16 \rangle\). When is the speed a minimum?

(6) Find equations of the normal plane and osculating plane of the curve of parametric equation \(\mathbf{r}(t) = \langle t, t^2, t^3 \rangle\) at the point \((2, 4, 8)\).

(7) Find the following limits if they exist
   \[
   \lim_{(x,y) \to (0,0)} \frac{x^4 - y^4}{x^2 + y^2}, \quad \lim_{(x,y) \to (0,0)} \frac{6x^3y}{2x^4 + y^4}
   \]

(8) Find the domain of definition of the following function and show it is differentiable on its domain:
   \[
   f(x, y) := e^x \cos(xy)
   \]
   Then find the linearization of this function at the point \((0, 0)\).

(9) The radius of a right circular cone is increasing at a rate of 1.8\(in/s\) while its height is decreasing at a rate of 2.5\(in/s\). At what rate is the volume of the cone changing when the radius is is 120 \(in\) and the height is 140 \(in\)?

(10) Find the dimensions of a rectangular box with largest volume if the sum of the lengths of its 12 edges is a constant \(c\).