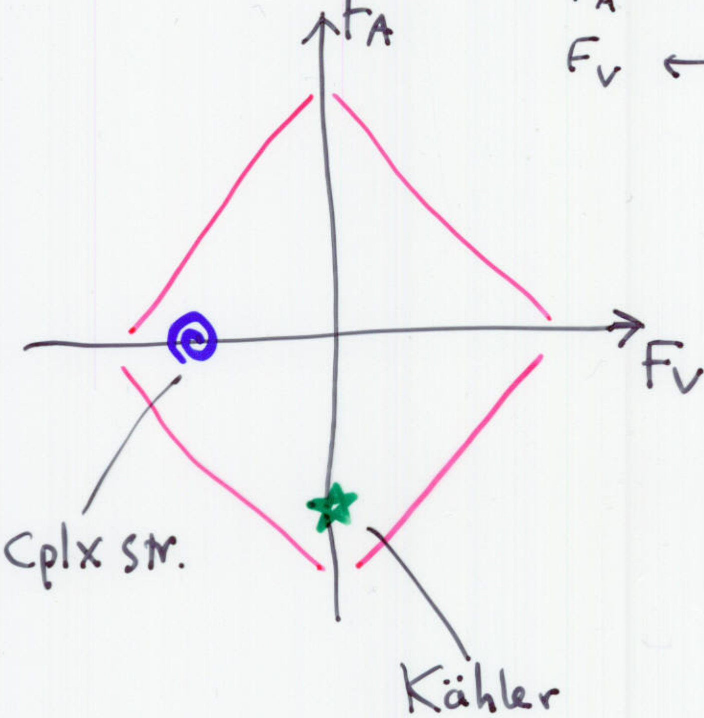
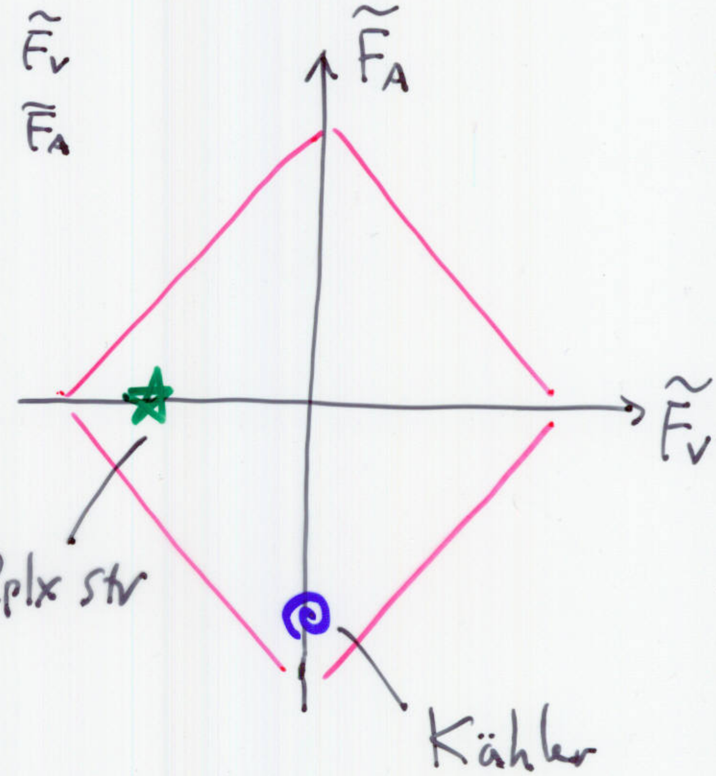
mirror

M (Calabi-Yau) $\longleftrightarrow$ $\tilde{M}$ (Calabi-Yau)

$F_A \longleftrightarrow \tilde{F}_V$

$F_V \longleftrightarrow \tilde{F}_A$





Calabi-Yau mirror pairs

Dixon, Lerche-Vafa-Warner : suggested existence

Greene-Plesser : non-geometric mode $\begin{pmatrix} \text{Gepner} \\ \cap \\ \text{LG orbifold} \end{pmatrix}$

$\downarrow$

Candelas-Lynker-Schimmrigk geometric

Batyrev

CY $\subset$ toric variety

hypersurface

Batyrev-Borisov

complete intersection of ..

Moduli space of (2,2) theories

$$\mathcal{M} = \underset{\substack{\uparrow \\ \text{chiral} \\ \text{deformations}}}{\mathcal{M}_c} \times \underset{\substack{\uparrow \\ \text{twisted chiral} \\ \text{deformations}}}{\mathcal{M}_t}$$

Decoupling theorem

A-model correlators …. holomorphic functions on $\mathcal{M}_t$
constant along $\mathcal{M}_c$

B-model correlators …. holomorphic functions on $\mathcal{M}_c$
constant along $\mathcal{M}_t$

NLσ-model on M (CY mfd)

$\mathcal{M}_c$ = moduli space of complex structure of M.

$$\left(\begin{array}{l} M : \text{CY 3-fold} \Rightarrow \text{“Special geometry”} \\ \quad \cdots \text{determined by period integrals} \\ \quad \text{of } \Omega \text{ on } H_3(M, \mathbb{Z}) \end{array} \right)$$

$$\mathcal{M}_t = \{\text{complexified Kähler class } [\omega - iB]\} \subset H^2(M; \mathbb{C}/2\pi i\mathbb{Z})$$

+ stringy quantum correction

+ analytic continuation ???

Use of mirror symmetry $M \leftrightarrow \tilde{M}$

$$\mathcal{M}_t(M) = \mathcal{M}_c(\tilde{M}) = \text{classical}$$

Example : quintic

$$M = \{ \underbrace{G(X_1, \dots, X_5)}_{\text{degree } 5} = 0 \} \subset \mathbb{CP}^4$$

$\xleftrightarrow{\text{mirror}}$ $\widetilde{M}$ = a resolution of the orbifold of

$$Z_1^5 + \cdots + Z_5^5 - 5\psi Z_1 \cdots Z_5 = 0 \quad \text{in } \mathbb{CP}^4$$

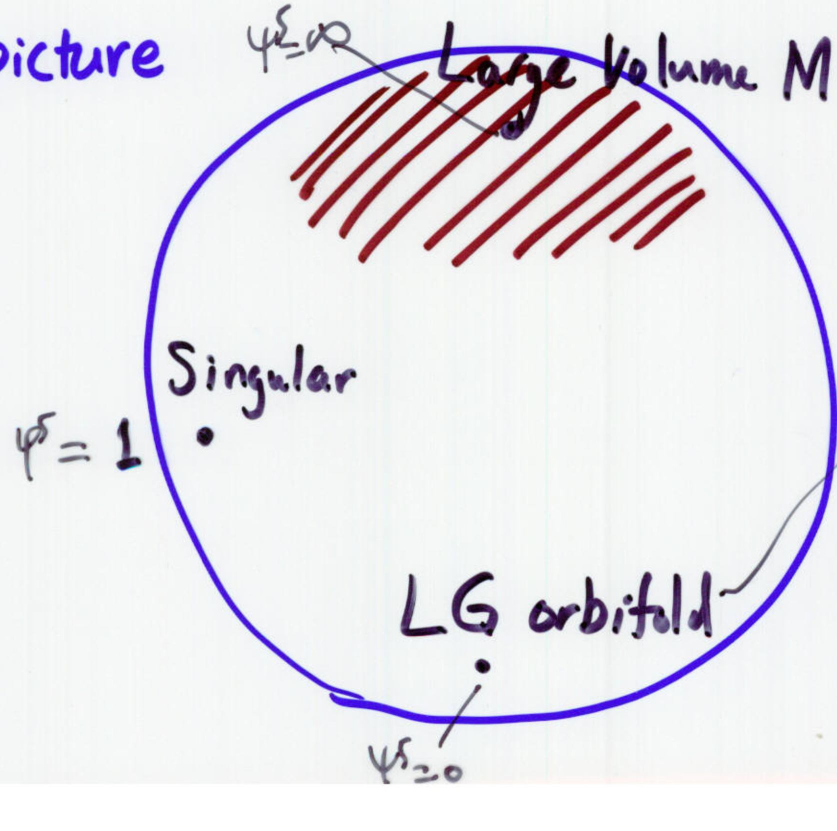
$$\text{by } (\mathbb{Z}_5)^3 : \; Z_i \to \omega_i Z_i \qquad \omega_i^5 = \omega_1 \cdots \omega_5 = 1$$

$$\mathcal{M}_t(M) = \mathcal{M}_c(\widetilde{M}) = \{ \psi^5 \}$$

Candelas, de la Ossa
Green and Parkes

3 special points $\psi^5 = \begin{cases} 0 & \widetilde{M} = \mathbb{Z}_5 \text{ symmetric} \\ 1 & \widetilde{M} : \text{conifold singularity (ODP)} \\ \infty & \widetilde{M} : \text{union of five } \mathbb{CP}^3\text{'s} \end{cases}$

picture



LG model with
$W = G(X_1, \dots, X_5)$
modulo $\mathbb{Z}_5 : X_i \to e^{2\pi i/5} X_i$

Linear Sigma Model

idea: $\phi_1, \ldots, \phi_N$: real scalar fields

$$L = \sum_{i=1}^{N} \frac{1}{2}\left\{ (\partial_t \phi_i)^2 - (\partial_\sigma \phi_i)^2 \right\} - \frac{\lambda^2}{2}\left(\sum_{i=1}^{N} \phi_i^2 - r \right)^2$$

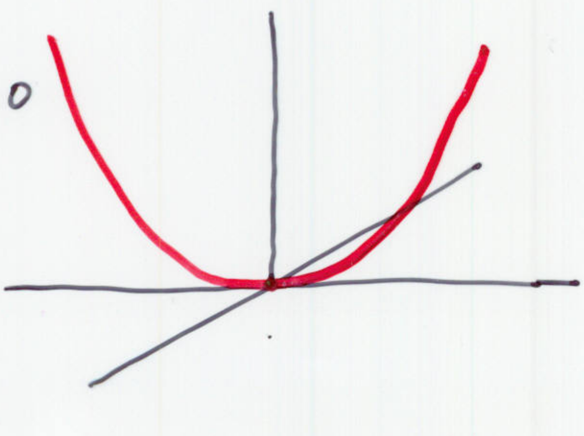
potential $\quad U(\phi) = \dfrac{\lambda^2}{2}\left(\displaystyle\sum_{i=1}^{N} \phi_i^2 - r \right)^2$

$r > 0$

At low energies, you are stuck at the bottom

.... NLσM on $M = S^{N-1}$

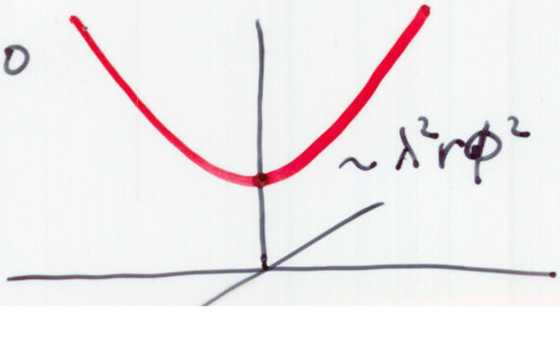
$r = 0$



ϕ_i are massless

$$U = \frac{\lambda^2}{2}\phi^4$$

$r < 0$



ϕ_i massive $\left(M = \lambda\sqrt{2r} \right)$

No low energy d.o.f.

LσM for projective space

$\phi_1, \ldots, \phi_N$ complex scalar fields

$A = A_t dt + A_\sigma d\sigma$ $\quad U(1)$ gauge field

$$L = \sum_{i=1}^{N}\left(|D_t\phi_i|^2 - |D_\sigma\phi_i|^2\right) + \frac{1}{2e^2}(F_{t\sigma})^2 - \underline{\frac{\lambda^2}{2}\left(\sum_{i=1}^{N}|\phi_i|^2 - r\right)^2}$$

$$D\phi_i = d\phi_i + iA\phi_i \quad , \quad F = dA \text{ (curvature)}$$

mod. out by $U(1)$ gauge symmetry $\begin{cases} \phi_i \to e^{i\gamma}\phi_i \\ A \to A - d\gamma \end{cases}$

$r > 0$: **bottom** $= \left\{ (\phi_1, \ldots, \phi_N) \,\middle|\, \sum_{i=1}^{N}|\phi_i|^2 = r \right\} / U(1)$

$$\cong (\mathbb{C}^N - 0)/\mathbb{C}^*$$

$$= \underline{\mathbb{CP}^{N-1}}$$

Other modes have mass $\sim e\sqrt{r}$ or $\lambda\sqrt{r}$

$E \ll e\sqrt{r}, \lambda\sqrt{r}$: NLσM on $\mathbf{M = \mathbb{CP}^{N-1}}$

$r = 0$: $U \sim |\phi|^4$, $r < 0$: massive.

... supersymmetric version :

ϕ_i ($\mathbb{C}$-scalar) $\xrightarrow{\text{add}}$ $\psi_{\pm i}$ (Dirac fermion)

A ($U(1)$-gauge f.) $\xrightarrow{\text{add}}$ $\lambda_\pm$ (Dirac fermion), σ ($\mathbb{C}$-scalar)

$$
\begin{aligned}
L = \sum_{i=1}^{N} \Big\{ & \underline{|D_0\phi_i|^2 - |D_1\phi_i|^2} - \underline{|\sigma|^2|\phi_i|^2} \\
& + i\bar{\psi}_{-i}(D_0+D_1)\psi_{-i} + i\bar{\psi}_{+i}(D_0-D_1)\psi_{+i} \\
& - \bar{\psi}_{-i}\sigma\psi_{+i} - \bar{\psi}_{+i}\bar{\sigma}\psi_{-i} \\
& - i\bar{\phi}_i\lambda_-\psi_{+i} + i\bar{\phi}_i\lambda_+\psi_{-i} + i\bar{\psi}_{+}\bar{\lambda}_-\phi_i - i\bar{\psi}_{-i}\bar{\lambda}_+\phi_i \Big\} \\
& + \frac{1}{2e^2}\Big\{ \underline{F_{01}^2} + |\partial_0\sigma|^2 - |\partial_1\sigma|^2 \\
& \qquad + i\bar{\lambda}_-(\partial_0+\partial_1)\lambda_- + i\bar{\lambda}_+(\partial_0-\partial_1)\lambda_+ \Big\} \\
& - \underline{\frac{e^2}{2}\Big(\sum_{i=1}^{N}|\phi_i|^2 - r\Big)^2} + \underbrace{\theta F_{01}}_{\text{Theta term}}
\end{aligned}
$$

$(t, \sigma) = (x^0, x^1)$

Superfield (encoding)

$$\Phi_i = \phi_i + \theta^+\psi_{+i} + \theta^-\psi_{-i} + \theta^+\theta^- f_i \quad + \text{derivatives}$$

auxiliary fields

$$\begin{aligned} V = {} & \theta^-\bar\theta^-(A_0 - A_1) + \theta^+\bar\theta^+(A_0 + A_1) \\ & - \theta^-\bar\theta^+\sigma - \theta^+\bar\theta^-\bar\sigma \\ & + i\theta^-\theta^+(\bar\theta^-\bar\lambda_- + \bar\theta^+\bar\lambda_+) + i\bar\theta^+\bar\theta^-(\theta^-\lambda_- + \theta^+\lambda_+) + \theta^+\theta^-\bar\theta^-\bar\theta^+ D \end{aligned}$$

$$\begin{aligned} L = {} & \int d\theta^+ d\theta^- d\bar\theta^- d\bar\theta^+ \left(\sum_{i=1}^N \bar\Phi_i e^V \Phi_i - \frac{1}{2e^2}\bar\Sigma\Sigma \right) \\ & + \mathrm{Re}\int d\theta^+ d\bar\theta^- \left(-t\Sigma \right)\Big|_{\bar\theta^+ = \theta^- = 0} \end{aligned}$$

$$\Sigma = \bar D_+ D_- V = \sigma + i\theta^+\bar\lambda_+ - i\bar\theta^-\lambda_- + \theta^+\bar\theta^-(D - iF_{01}) + \cdots$$

⋯ super curvature

$$t = r - i\theta \longleftrightarrow \text{complexified Kähler class}$$

Generalization —— M: toric mfd

$$\phi_i \to e^{i\sum_{a=1}^k Q_i^a \gamma_a}\phi_i \qquad Q_i^a \cdots \text{charge}$$

$$\begin{aligned} L = {} & \int d^4\theta \left(\sum_{i=1}^N \bar\Phi_i e^{\sum_{a=1}^k Q_i^a V_a}\Phi_i - \sum_{a=1}^k \frac{1}{2e_a^2}\bar\Sigma_a\Sigma_a \right) \\ & + \mathrm{Re}\int d\theta^+ d\bar\theta^- \left(-\sum_{a=1}^k t^a \Sigma_a \right)\Big|_{\bar\theta^+ = \theta^- = 0} \quad \Rightarrow M = (\mathbb{C}^N - \text{bad})/(\mathbb{C}^*)^k \end{aligned}$$

Degree d hypersurface in $\mathbb{CP}^{N-1}$

$\Phi_1, \dots, \Phi_N$: charge 1

P : charge $-d$.

$$L = \int d\theta^+ d\theta^- d\bar\theta^- d\bar\theta^+ \left(\sum_{i=1}^N \bar\Phi_i e^V \Phi_i + \bar P e^{-dV} P - \frac{1}{2e^2} \bar\Sigma \Sigma \right)$$

$$+ \operatorname{Re} \int d\theta^+ d\bar\theta^- \left(-t\Sigma \right)\Big|_{\bar\theta^+ = \theta^- = 0}$$

"twisted superpotential"

$$+ \operatorname{Re} \int d\theta^+ d\theta^- \left(P \underbrace{G(\Phi_1, \dots, \Phi_N)}_{\text{degree } d \text{ Polynomial}} \right)$$

superpotential

(assume $\{G(\phi) = 0\} \subset \mathbb{CP}^{N-1}$ is <u>smooth</u>)

<u>Potential</u>

$$U = |\sigma|^2 \left(\sum_{i=1}^N |\phi_i|^2 + 5^2 |p|^2 \right)$$

$$+ \frac{e^2}{2} \left(\sum_{i=1}^N |\phi_i|^2 - 5|p|^2 - r \right)^2$$

$$+ |G(\phi_1, \dots, \phi_N)|^2$$

$$+ \sum_{i=1}^N |p|^2 \left| \frac{\partial G(\phi)}{\partial \phi_i} \right|^2$$

$r > 0 \quad U = 0 \Rightarrow \phi_i \neq 0 \Rightarrow \sigma = 0,\ p = 0$

bottom of U $= \left\{ \phi \;\middle|\; G(\phi) = 0,\ \sum_{i=1}^{N} |\phi_i|^2 = r \right\} / U(1)$

$= \left\{ \phi \neq 0 \mid G(\phi) = 0 \right\} / \mathbb{C}^*$

$=$ hypersurface $G(\phi) = 0$ in $\mathbb{CP}^{N-1}$.

$r < 0 \quad U = 0 \Rightarrow p \neq 0,\ \sigma = 0,\ \phi_i = 0$ a point !

massless fields : $\phi_1, \ldots, \phi_N$

$p \neq 0$ breaks $U(1)$ gauge symmetry to $\mathbb{Z}_d \subset U(1)$.

low energy theory = **LG model with**

$W = G(\phi_1, \ldots, \phi_N)$ **modulo** $\mathbb{Z}_d$

$\phi_i \to \omega \phi_i \quad \omega^d = 1$

… **Landau-Ginzburg Orbifold**

$r = 0 \quad U = 0 \Rightarrow \phi_i = p = 0,$ σ is unconstrained … non-compact

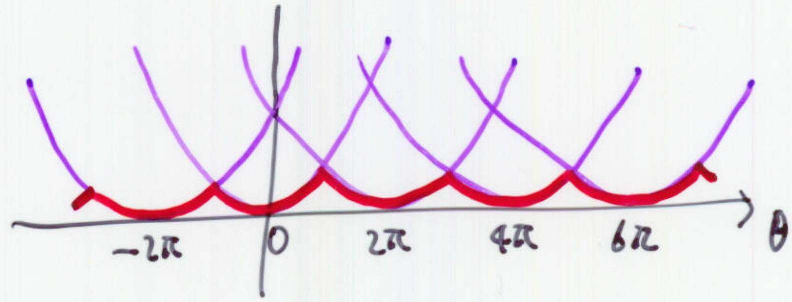
$\longrightarrow$ signals a singularity

Large σ-region

from $\frac{1}{2e^2}F_{01}^2 + \theta F_{01}$

$$\underline{E_{vac}} = \frac{e^2}{2} r^2 + \frac{e^2}{2}\hat{\theta}^2 = \frac{e^2}{2}|\hat{t}|^2$$

$$|\hat{\theta}| = \min_{n\in\mathbb{Z}} |\theta + 2\pi n|$$

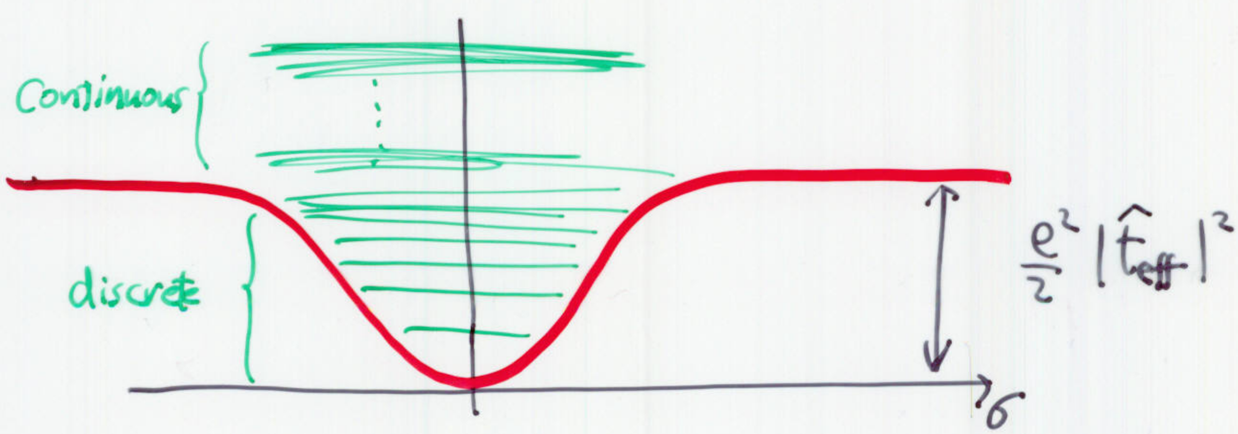


In quantum theory, t is corrected to $t_{eff}(\sigma)$

$$t_{eff}(\sigma) = t + \sum_i Q_i \log(Q_i\sigma)$$

$$= t + N\log\sigma - d\log(-d\sigma)$$

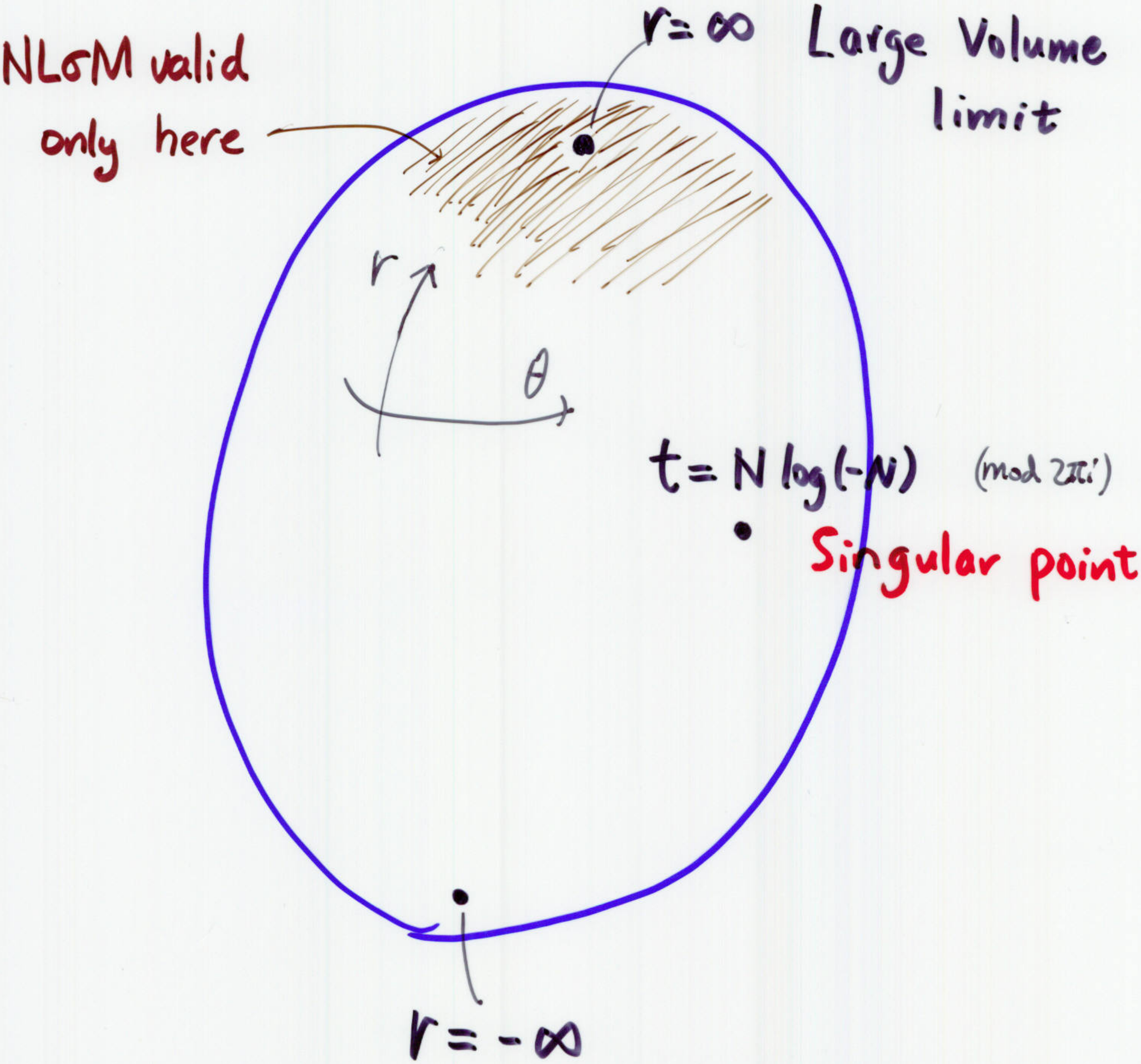
$d = N$ ($M = CY$): $\quad t_{eff} = t - N\log(-N)$



$t_{eff} = 0$: **Singularity**

mod $2\pi i\mathbb{Z}$

The moduli space $\mathcal{M}_t$



LG orbifold

$$W = G(\phi_1, \ldots, \phi_N) \;/\; \mathbb{Z}_{d=N}$$

$d \neq N$ cases :

r changes as the energy scale μ is changed

$$r_{\text{eff}}(\mu) = \mathrm{Re}\, t_{\text{eff}}(\mu)$$

$d < N$ (M : Fano)

$$\text{high } \mu \longrightarrow \text{low } \mu$$

$$r_{\text{eff}} \gg 0 \longrightarrow r_{\text{eff}} \ll 0$$

NLσM on M $\longrightarrow$ LG orbifold $W = G/\mathbb{Z}_d$

+ $(N-d)$ massive vacua

at $t_{\text{eff}}(\sigma) = 0$

$d > N$ (M : general type)

$$\text{high } \mu \longrightarrow \text{low } \mu$$

$$r_{\text{eff}} \ll 0 \longrightarrow r_{\text{eff}} \gg 0$$

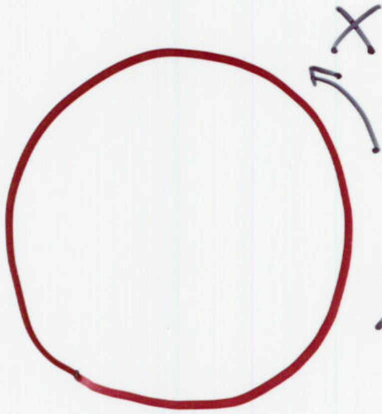
LG orbifold $W = G/\mathbb{Z}_d$ $\longrightarrow$ NLσM on M

+ $(d-N)$ massive vacua

at $t_{\text{eff}}(\sigma) = 0$.

T-duality

Sigma model on $M = S^1$, radius R



$X \equiv X + 2\pi R$

a string state is specified by $\begin{cases} \cdot \text{ momentum} \\ \cdot \text{ winding \#} \\ \cdot \text{ oscilation \#'s} \end{cases}$

- Momentum p = eigenvalue of $-i\frac{\partial}{\partial X}$

  $\leftrightarrow$ wave function e^{ipX} .... single valued $\left(\begin{matrix}\text{invariant} \\ X \to X + 2\pi R\end{matrix}\right)$

  iff $\boxed{p = \frac{\ell}{R} \quad (\ell \in \mathbb{Z})}$ quantized

- Winding # m : Contribution to the mass is

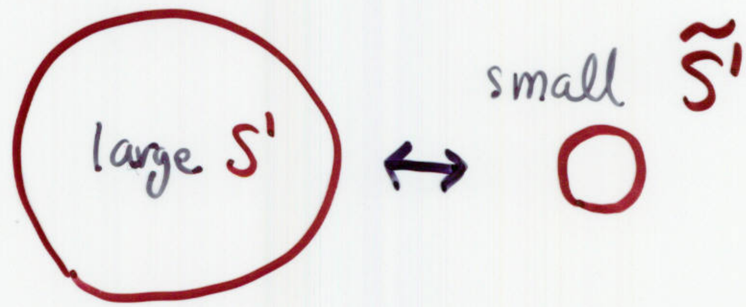
$$M = \text{tension} \times \text{length} = \frac{2\pi R m}{2\pi\alpha'} = \frac{Rm}{\alpha'}$$

$$E^2 = p^2 + M^2$$

$$= \left(\frac{\ell}{R}\right)^2 + \left(\frac{Rm}{\alpha'}\right)^2 + \underbrace{M_{osc}^2}_{\text{indep of } R}$$

This is invariant under

$$R \rightarrow \frac{\alpha'}{R}$$



$$\ell \longrightarrow m$$

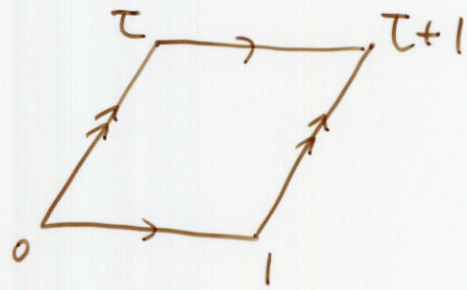
$$m \longrightarrow \ell$$

momentum ↔ winding #

String theory on $(S', R) \cong$ String theory on $\left(\tilde{S}', \frac{\alpha'}{R}\right)$

this is called **T-duality**

$M = T^2$

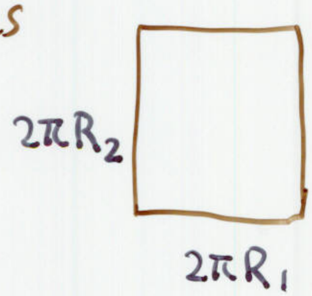


Complex structure τ

Complexified Kähler class $\rho = \frac{1}{2\pi i}\int_{T^2}\left(-\frac{\omega}{2\pi\alpha'} + iB\right)$

$$= i\,\frac{\text{Area}}{4\pi^2\alpha'} + \int_{T^2}\frac{B}{2\pi}$$

Square torus



$$\tau = \frac{R_2}{R_1}\,i$$

$$\rho = \frac{R_1 R_2}{\alpha'}\,i$$

T-duality on α' : $(R_1, R_2) \to \left(\frac{\alpha'}{R_1}, R_2\right)$

$$\tau = \frac{R_2}{R_1}\,i \longrightarrow \tilde{\tau} = \frac{R_2 R_1}{\alpha'}\,i = \rho$$

$$\rho = \frac{R_1 R_2}{\alpha'}\,i \longrightarrow \tilde{\rho} = \frac{R_2}{R_1}\,i = \tau$$

Complex structure $\longleftrightarrow$ Kähler class

Mirror Symmetry !

$\rightsquigarrow$ Mirror Symmetry is T-duality along middle dim. torus fibrations

- Strominger - Yau - Zaslow

  SLAG T^3 fibrations for CY^3 – use D-branes

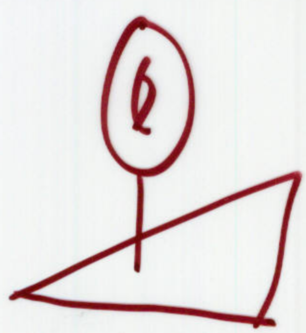
  $\rightsquigarrow$ mathematical works

  M. Gross, WD Ruan, ... I. Zarkov,

  ... B. Siebert - Gross, Kontsevich - Soibelman, ...

  (stay in geometry)

- Toric Variety



Fendley - Intriligator

Givental

⋮

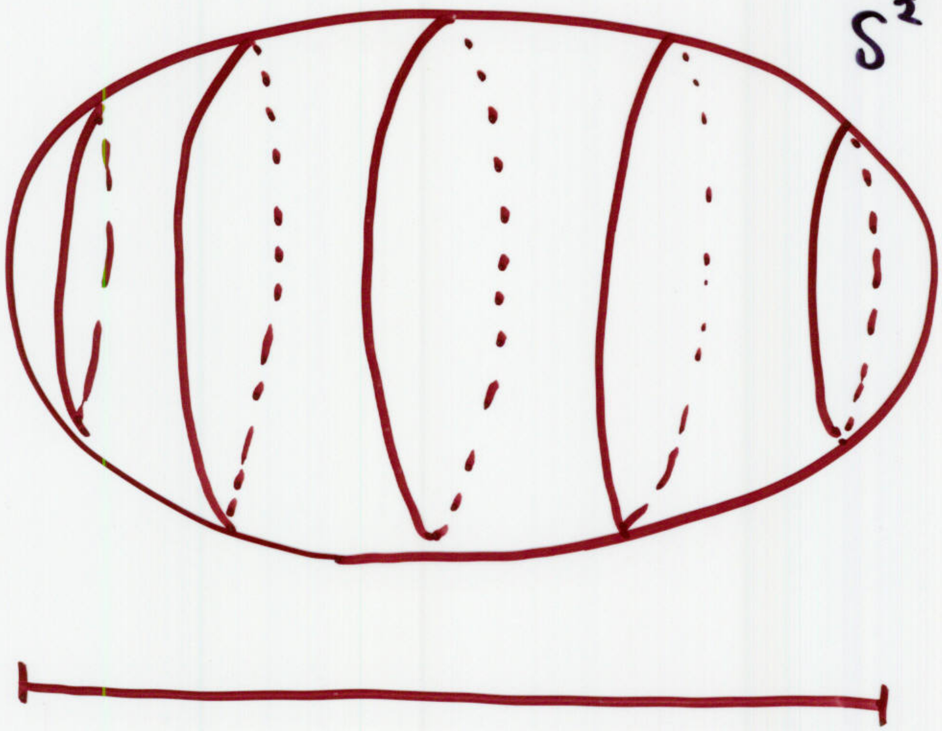
H. Vafa

use $L\sigma M$

(go out of geometry)

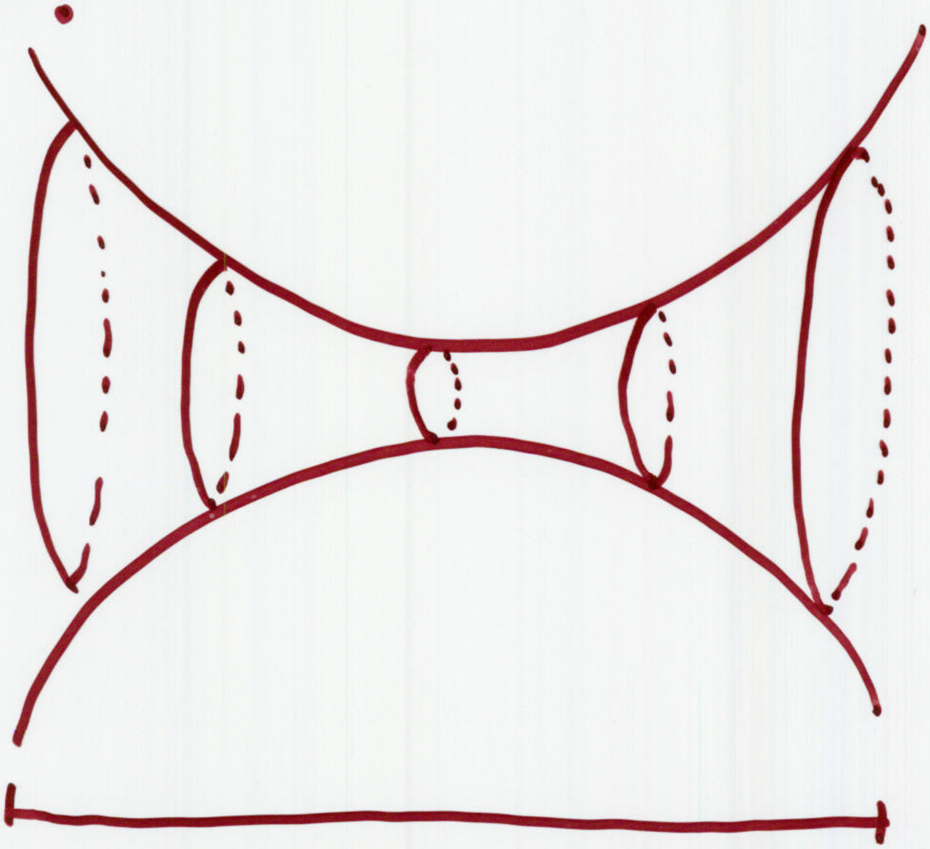
$M = S^2$

$$t = \int_{S^2} \left(\frac{\omega}{2\pi\alpha'} - iB \right)$$



- momentum along fibre conserved
- winding # 〃 NOT conserved
- F_V conserved
- F_A anomalous
- Compact

$\tilde{M} = ?$



- Winding # along fibre conserved ✓
- momentum " conserved ?
- F_A conserved ✓
- F_V conserved ?
- non-compact ?

True Story :

The mirror is a LG model

$$\boxed{\begin{aligned} \tilde{M} &= \mathbb{C}^* = \{ Y \equiv Y + 2\pi i \} \\ \tilde{W} &= e^{-Y} + e^{-t+Y} \end{aligned}}$$

- translation symmetry along fibre ($\mathrm{Im}\,Y$-shift) is broken by $\tilde{W}$ ✓
- F_V not conserved by $\tilde{W}$ (which is not quasi-homogeneous) ✓
- Potential $U = |\partial_Y \tilde{W}|^2 = |e^{-Y} - e^{-t+Y}|^2$ effectively compactifies the theory ✓

$|e^{-Y}|$: wall at $\mathrm{Re}\,Y \to -\infty$, $|e^{-t+Y}|^2$ wall at $\mathrm{Re}\,Y \to +\infty$.

Derivation

H-V

Use LσM

$$\mathcal{L} = \sum_{i=1}^{2} \left\{ |D_0 \phi_i|^2 - |D_1 \phi_i|^2 + \cdots \right\}$$
$$+ \frac{1}{2e^2} \left\{ (F_{01})^2 + \cdots \right\}$$
$$- \frac{e^2}{2} \left(|\phi_1|^2 + |\phi_2|^2 - r \right)^2 + \theta F_{01}$$

S^1 fibres $\longleftrightarrow$ $\arg(\phi_1)$, $\arg(\phi_2)$

modulo gauge transf. (common shift)

Idea : T-dualize $\arg(\phi_1)$ & $\arg(\phi_2)$

$$\boxed{S^1 \text{ case}}$$

$$B \in \Omega^1(\Sigma)$$

$$S' = \int \frac{1}{2R^2}|B|^2 - i\, B \wedge d\varphi \qquad \varphi : \Sigma \to S^1_{2\pi} \quad \varphi \equiv \varphi + 2\pi$$

$$\begin{tikzcd}[column sep=small] & S' \arrow[dl, "\int \mathcal{D}B"', "B = -iR^2 * d\varphi"] \arrow[dr, "\int \mathcal{D}\varphi", "B = d\tilde{\varphi} \quad \tilde{\varphi} \equiv \tilde{\varphi} + 2\pi"'] & \\ S = \int \frac{R^2}{2}|d\varphi|^2 & & \tilde{S} = \int \frac{1}{2R^2}|d\tilde{\varphi}|^2 \end{tikzcd}$$

$\uparrow$ Sigma model on (S^1, R) $\overset{T}{\longleftrightarrow}$ $\uparrow$ Sigma model on $(S^1, \frac{1}{R})$

$$-iR * d\varphi = \frac{1}{R} d\tilde{\varphi}$$

momentum $\longleftrightarrow$ winding #

*: Winding # $\longleftrightarrow$ momentum

defect $\dot{x}$ $\quad \varphi \to \varphi + 2\pi$ $\quad \overset{T}{\longleftrightarrow}$ $\quad \cdot \exp(i\tilde{\varphi}(x))$

... momentum creation at x

U(1) theory with a single Φ (charge 1):

$\phi = \rho e^{i\varphi}$ T-dualize φ

$$|D_\mu \phi|^2 = (\partial_\mu \rho)^2 + \rho^2(\partial_\mu \varphi + A_\mu)^2$$

$$S' = \int \frac{1}{4\rho^2}|B|^2 + \int i B \wedge (d\varphi + A)$$

$\int \mathcal{D}B$ $\swarrow$ $\searrow$ $\int \mathcal{D}\varphi$ $B = d\tilde{\varphi}$

$$S = \int \rho^2 |d\varphi + A|^2 \qquad \tilde{S} = \int \frac{1}{4\rho^2}|d\tilde{\varphi}|^2 - \int i\, d\tilde{\varphi}\wedge A$$

$$\int i\, d\tilde{\varphi}\wedge A = i\int \tilde{\varphi} F_A$$

Dynamical Theta angle

recall:

$t = r - i\theta$ enters into $\int d\theta^+ d\bar{\theta}^- (-t\Sigma)\big|_{\bar{\theta}^+ = \theta^- = 0}$

$$\tilde{W} = -t\Sigma \xrightarrow{\text{T-dual}} (Y - t)\Sigma$$

$Y = \underbrace{\rho^2 - i\tilde{\varphi}}_{\text{twisted chiral}} + \dots$

$\operatorname{Im} Y \overset{T}{\longleftrightarrow} \arg \Phi$

$\operatorname{Re} Y = \bar{\Phi} e^V \Phi$

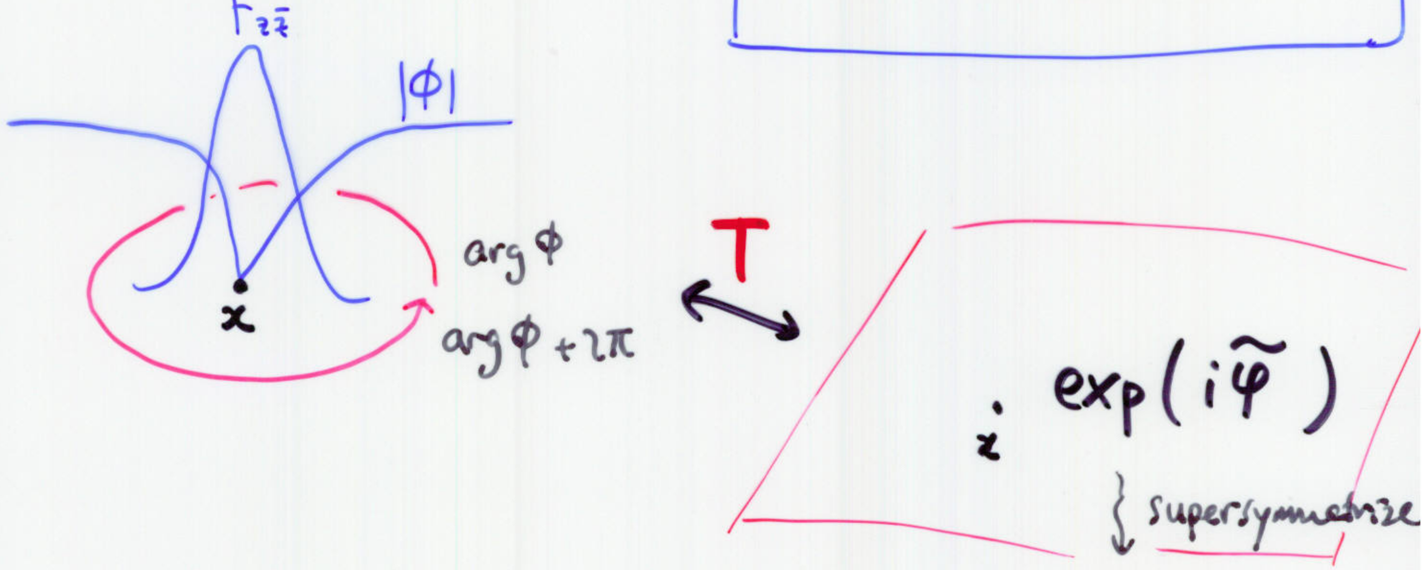
(ϕ, A) system with

$$\mathcal{L} = |D_\mu \phi|^2 + \frac{e^2}{2}\left(|\phi|^2 - r\right)^2 + \frac{1}{2e^2}(F_{\mu\nu})^2$$

has <u>vortex configuration</u>

preserving $\bar{Q}_+ , Q_-$

$$\boxed{\begin{aligned} &D_{\bar z}\phi = 0 \\ &|\phi|^2 - r + \frac{i}{e^2}F_{z\bar z} = 0 \end{aligned}}$$



$\exp(-Y)$

This generates a term e^{-Y}

in the twisted superpotential

$$\underline{\widetilde{W}_{dual} = (Y-t)\Sigma + e^{-Y}}$$

... exact result

For S^2 : two charged fields $\Phi_1, \Phi_2 \longrightarrow Y_1, Y_2$

$$\widetilde{W} = (Y_1 + Y_2 - t)\Sigma + e^{-Y_1} + e^{-Y_2}$$

Σ has mass $\sim e^2$: integrate out at $E \ll e^2$.

$\Rightarrow$ Constraint $Y_1 + Y_2 - t = 0$

$$\therefore \ \underline{\widetilde{W} = e^{-Y_1} + e^{-t+Y_1}}$$

For (other) toric variety $(\mathbb{C}^N - \text{bad})/(\mathbb{C}^*)^k$ $\quad \Phi_i \to e^{iQ_i^a r_a}\Phi_i$

$$\widetilde{W} = \sum_{a=1}^{k}\left(\sum_{i=1}^{N} Q_i^a Y_i - t^a\right)\Sigma_a + e^{-Y_1} + \cdots + e^{-Y_N}$$

$$\xrightarrow{\text{int-out } \Sigma_a} \boxed{\begin{aligned} \widetilde{M} &= \left\{ (Y_1, \cdots, Y_N) \;\middle|\; \begin{array}{l} Y_i \equiv Y_i + 2\pi i \\ \sum_{i=1}^{N} Q_i^a Y_i = t^a \end{array} \right\} = (\mathbb{C}^*)^{N-k} \\ \widetilde{W} &= e^{-Y_1} + \cdots + e^{-Y_N} \end{aligned}}$$

Degree d hypersurface in $\mathbb{C}P^{N-1}$

$$\begin{array}{l} \Phi_1, \dots, \Phi_N, \; P \\ 1, \dots, 1 \;, -d \end{array} \qquad W = P\, G(\Phi_1, \dots, \Phi_N)$$

Ignore W $\Rightarrow$ dualize Φ_i, P $\to$ $Y_1, \dots, Y_N, Y_P$

$$\text{mirror} = \begin{cases} Y_1 + \cdots + Y_N - d\,Y_P = t \\ \widetilde{W} = e^{-Y_1} + \cdots + e^{-Y_N} + e^{-Y_P} \end{cases}$$

define Z_i by $Y_i = d \cdot Z_i$, $Y_P = Z_1 + \cdots + Z_N - \frac{t}{d}$

$$(e^{-Y_i}, e^{-Y_P}) \leftarrow e^{-Z_i}$$

$$1 \quad : \quad \mathbb{Z}_d^{N-1} \qquad e^{-Z_i} \to \omega_i e^{-Z_i} \qquad \omega_i^d = \omega_1 \cdots \omega_N = 1$$

LG orbifold $\quad \widetilde{W} = (e^{-Z_1})^d + \cdots + (e^{-Z_N})^d + e^{t/d} e^{-Z_1} \cdots e^{-Z_N}$

$$\text{mod } \mathbb{Z}_d^{N-1}$$

Effect of W : breaks phase sym of P, Φ_i

$\Leftrightarrow$ non-conservation of winding # in Y_P, Y_i

$\rightsquigarrow$ Change of variables . $X_i = e^{-Z_i}$ $\mathbb{C}$-valued.

LG orbifold $\quad \widetilde{W} = X_1^d + \cdots + X_N^d + e^{t/d} X_1 \cdots X_N$

$$\text{mod } \mathbb{Z}_d^{N-1} \; : X_i \to \omega_i X_i \quad \omega_i^d = \omega_1 \cdots \omega_N = 1$$