

Universal spaces in birational geometry — Fedor Bogomolov, October 8, 2010

I want to discuss our joint results with Yuri Tschinkel. The Bloch-Kato conjecture implies that cohomology elements with finite constant coefficients of an algebraic variety can be induced from abelian quotient of the fundamental group after restriction to some open subvariety. Our theorem on the structure of the Galois groups of functional fields implies a similar result for nonramified cohomology. Namely for any element a of nonramified cohomology $H_{nr}^i(\text{Gal}(\bar{K}/K), \mathbb{Z}_l^n, i \geq 2, K = \bar{F}_p(X), p \neq l)$ there is a finite topological quotient G^c of $\text{Gal}(\bar{K}/K)$ such that a is induced from a nonramified element b of $H_{nr}^i(G^c, \mathbb{Z}_l^n)$. Here G^c is a finite group which is a central extension of an abelian group and the element b is induced from the abelian quotient of G_c . It has the following geometric interpretation: there exists a rational map $f : X \rightarrow \prod P^i/A$ and a nonramified element $b \in H_{nr}^i(G^c, \mathbb{Z}_l^n)$ such that $f^*(b) = a$. Here $A = G^c/[G^c, G^c]$ acts projectively on each P^i and generically freely on the product. In this case the image of f intersects an open subvariety of smooth points in $\prod P^i/A$ (in order for the map of nonramified cohomology to be defined). Thus the varieties of type $\prod P^i/A$ constitute a class of universal spaces varieties for nonramified cohomology of varieties over \bar{F}_p . The situation is more interesting in the case of \bar{Q} . It brings a number of questions which I also plan to discuss in the talk. If time permits I will also discuss the proof of the Grothendieck section conjecture for the groups G_c as above.