Problem 1. The aim of this problem is to give a couple alternative approaches to the Euler equation

\[ x^2y'' + \alpha xy' + \beta y = 0. \]  

(i) Suppose that the indicial equation

\[ r^2 + (\alpha - 1)r + \beta = 0, \]

has a repeated root \( r = r_1 \). By what we discussed in class, we obtain a solution \( y_1(x) = x^{r_1} \). Use reduction of order to find a function \( v(x) \) so that \( y_2 = v(x)x^{r_1} \) is second solution of (1) with the property that \( \{y_1, y_2\} \) is a fundamental set of solutions.

(ii) Suppose \( y(x) \) is a solution of (1). Define a new variable \( t \) by the change of variables formula \( t = \ln(x) \). Find the differential equation in the variable \( t \) that \( \phi(t) := y(x(t)) \) satisfies. (Hint: the equation you find should be very familiar to you).

(iii) Explain why the equation you found in part (ii) above trivializes everything we said in class about the Euler equation. That is, write down the general solution to equation (1) in the case that the indicial equation has real distinct, repeated, or complex roots using only the theory of constant coefficient linear equations of order 2.

• §5.5 # 3, 7, 12

Wednesday, June 6

• §5.6 # 1, 14, 19 (a)-(d), 20
• Review §1.2, 1.3, 2.1, 2.2, 2.4-2.6, 3.1-3.6, 4.1-4.4, 5.1-5.6 for the midterm!