An open problem in nonlinear dispersive equations is the asymptotic completeness conjecture which states that any initial data eventually converges to a superposition of coherent states. A big obstacle in solving the conjecture is the fact that the set of coherent states is not completely known. I will present recent results which make significant progress in finding all coherent states (solitary waves) in nonlinear Schrödinger equation.

The presentation will not assume any prior knowledge of bifurcation theories or nonlinear Schrödinger equation. But it will show how the Hamiltonian structure and the symmetries of the equation can be transformed into ordinary differential inequalities for certain norms along branches of solitary waves, how the differential inequalities give the asymptotic behavior and the appropriate renormalization for branches approaching the boundary of the bifurcation diagram, and how renormalization, concentration compactness, and bifurcations from infinity identify all possible limit points on the boundary of the bifurcation diagram. The global bifurcation picture is then determined by how the branches coming from the boundary intersect in the interior of the bifurcation diagram, which, in turn, is given by the geometry of analytical varieties near singularity points. This is joint work with Vivek Natarajan (U. of Illinois).