MID TERM II
CALCULUS III (98599 SEC. 10)

- The use of class notes, book, formulae sheet, calculator is not permitted.
- In order to get full credit, you must show all your work.
- You have one hour and fifteen minutes.
- Do not forget to write your name and UNI in the space provided below.

Print UNI ______________________
Print Name _____________________

For Grader’s use only:

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Problem 1 (15 points) Consider the following function
\[ f(x, y, z) = x\sqrt{y^2 + z^2} \]

(a) Compute the differential of \( f \).

(b) Find the linear approximation of \( f \) at \((2, 3, 4)\).

(c) Use the linear approximation to find an approximate value of \( f(1.98, 3.01, 3.99) \).
Problem 2 (20 points) Find the equation of the tangent plane to a surface $S$ which contains the following two parametric curves, at the point of intersection of these curves.

\begin{align*}
\vec{r}_1(t) &= \langle t - 2, (t - 1)^2, 5 \rangle \\
\vec{r}_2(s) &= \langle \cos \left( \frac{\pi}{4} s \right), \sin \left( \frac{\pi}{4} s \right), s^2 + 1 \rangle
\end{align*}
Problem 3 (15 points) Prove that the following two surfaces have the same tangent planes at \((1, -1, 0)\)

\[
x^2z + yz + 2xy + 2 = 0
\]

\[
x^2 + y^2 + z^2 - 2 = 0
\]
Problem 4 (20 points) True/False. Justify your answers with a proof or a counterexample.

(a) For two parameteric curves \( \vec{r}_1(t) \) and \( \vec{r}_2(t) \) we have
\[
\frac{d(\vec{r}_1(t) \times \vec{r}_2(t))}{dt} = \vec{r}_1'(t) \times \vec{r}_2'(t)
\]

(b) \[
\lim_{(x,y) \to (0,0)} (x^2 + y^2) \ln(\sqrt{x^2 + y^2}) = 0.
\]

(c) The line joining the center of a sphere and a point \( P \) on the sphere is always perpendicular to the tangent plane to the sphere at \( P \).

(d) Assume that \( f(x, y) \) is a symmetric function of two variables, that is, \( f(x, y) = f(y, x) \). Then \( f_x(x, y) \) is also symmetric.
Problem 5 (15 points) One side of a triangle is increasing at a rate of 3 cm/sec and a second side is decreasing at a rate of 2 cm/sec. Assume that the area of the triangle remains unchanged, at what rate is the angle between the two sides changing, when the first side is 20 cm, the second side is 30 cm and the angle is $\pi/6$. 
Problem 6 (15 points) Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) if

\[
yz + x \ln(y) - z^2 = 0
\]