The use of class notes, book, formulae sheet, calculator is not permitted.

In order to get full credit, you **must** show all your work.

You have **two hours and fifty minutes**.

Do not forget to write your name and UNI in the space provided below.

Print UNI ______________________
Print Name ______________________

For Grader’s use only:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>15</td>
<td>12</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td>15</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>
Problem 1 (20 points).

(a) Find the parametric equations describing the tangent line to the following parametric curve at $(0, 0, 1)$.

\[ \vec{r}(t) = (t \cos(2t), \sin(2t), e^t) \]

(b) Write the following complex number in the form $a + bu$.

\[ \left( \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \right)^{10} \]

(c) Write the equation of the tangent plane to the following surface at $(0, 1, 3)$.

\[ z^2 \cos(xy) - 3yz = x^2 \]
(d) Find the length of the following curve
\[ \vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle ; \quad 0 \leq t \leq 1 \]

(e) Evaluate the following limit, if it exists, or prove that it does not exist.
\[ \lim_{(x,y) \to (0,0)} \frac{\ln(1 + x^2 + y^2)}{x^2 + y^2} \]
Problem 2 (15 points). Let $C$ be the curve of intersection of the following two surfaces:

\[ x^2 + y^2 = 1 \]
\[ z = 3 - 2x^2 - 4y^2 \]

Find points on $C$ which are closest to and farthest from the origin.
Problem 3 (12 points). Consider the function 

\[ F(u, v, w) = \frac{1 - u^2}{u^2 + v^2 + w^2} \]

(a) From point \((0, 1, 1)\), in which direction the function increases fastest? What is the fastest rate of increase?

(b) What is the rate of change of \(F\) along the direction from \((0, 1, 1)\) towards \((2, 2, 2)\)?

(c) Use the linear approximation of \(F\) to find its approximate value at \((0.1, 0.9, 0.9)\).
Problem 4 (12 points).  A projectile is fired with an initial speed of 100 \text{m/s} at an angle of 60 \degree.

(a) Write the position \( \vec{r}(t) \) and velocity \( \vec{v}(t) \) of the particle at time \( t \).

(b) At what times is the projectile at the height three quarters of its maximum height?
Problem 5 (8 points). Assume $z$ is implicitly defined as a function of $x$ and $y$ by:

$$\cos(yz) + x^2z = 9$$

If at $x = 2$, $y = 0$, $z = 2$, the value of $x$ starts increasing at the rate of 1 unit per second, and the value of $y$ starts decreasing at the rate of 2 units per second, compute the rate of change of $z$. 
Problem 6 (8 points). Let \( x(t) = e^{rt} \), where \( r \) is a constant.

(a) Compute \( x'(t) \) and \( x''(t) \).

(b) Find the (complex) values of \( r \) for which the function \( x(t) \) satisfies:
\[
x''(t) + x'(t) + x(t) = 0
\]
Problem 7 (15 points). Find the absolute maximum and minimum values of 

\[ f(x, y) = xy - \frac{5}{3}x^3 + 4 \]

on the domain \( D \) bounded between two parabolas \( y = x^2 \) and \( y = 8 - x^2 \).
Problem 8 (10 points). Let $\vec{r}(t) = \langle f(t), g(t) \rangle$ be a parametric curve in $\mathbb{R}^2$.

(a) Obtain an expression for the function $\phi(t) = \text{angle between } \vec{r}'(t) \text{ and the positive } x\text{-axis}.$

(b) Compute $\frac{d\phi}{dt}$.

(c) If $\kappa(t)$ is the curvature of $\vec{r}(t)$, verify the following equation:

$$\left| \frac{d\phi}{dt} \right| = |\vec{r}'(t)|\kappa(t)$$