1. **Cylindrical surfaces.**

These surfaces contain (and consist of) lines parallel to a given one. For example,

\[ y = x^2 \]

→ consists of lines parallel to \( z \)-axis

Example: \( x^2 + z^2 = 4 \)

→ consists of lines parallel to \( y \)-axis

Circle of radius 2
2. Review of conic sections

- Parabola

\[ y^2 = 4ax \] \( (a > 0) \)

\[ y^2 = -4ax \] \( (a < 0) \)

Parabola = set of points whose distance from a fixed point (focus) is same as distance from a fixed line (directrix)

\[ y = ax^2 \] \( (a > 0) \)
Ellipse.

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

\[ c^2 = a^2 - b^2 \quad a > b \]

\[ b > a; \quad c^2 = b^2 - a^2 \]

Ellipse = set of points the sum of whose distances from two fixed points (called foci) is constant.

Note: for \( a = b \), we get a circle.

Hyperbola.

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

\[ c^2 = a^2 + b^2 \]
Hyperbola = set of points the difference of whose distance from two fixed points is constant.

3. Quadratic surfaces.

\[ z = \frac{x^2}{4} + \frac{y^2}{9} \]

for a fixed value of \( z (= k^2) \) we get

\[ \frac{x^2}{(2k)^2} + \frac{y^2}{(3k)^2} = 1 \]

\( \rightarrow \) an ellipse

For a fixed value of \( y \), for instance \( y=0 \), we get

\[ z = \frac{x^2}{4} \]
Sketch of the surface

→ elliptic paraboloid

**Traces of a surface:** intersection curve of the surface \( S \) and a plane parallel to coordinate planes

For example, \( z = \frac{x^2}{4} + \frac{y^2}{9} \)

- **X-trace,**
  
  \( x = 0 \), \( z = \frac{y^2}{9} \)

  \( x = \pm 1 \), \( z = \frac{y^2}{9} + 1 \)

- **Z-traces,**
  
  \( z = 1 \): \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)

  \( z = 4 \): \( \frac{x^2}{16} + \frac{y^2}{36} = 1 \)
Example 2.
Use traces to sketch \( x^2 + \frac{y^2}{4} + z^2 = 1 \) (ellipsoid)

\[ z = y^2 - x^2 \]

\( x \)-trace

\[ z \]

\( y \)-trace

\[ z \]

\( z \)-trace

\[ z \]
More examples

\[ z^2 = x^2 + \frac{y^2}{4} \quad \text{(cone)} \]

\[ x^2 + \frac{y^2}{4} - z^2 = 1 \quad \text{(hyperboloid)} \]

\[ -x^2 - y^2 + z^2 = 4 \]