

## W4043 HOMEWORK #5

DUE: TUESDAY, FEBRUARY 27TH, 2007

The goal of this homework is to generalise our work on cyclotomic extensions for a prime  $p$  to the general  $n \in \mathbb{N}$ .

- (1) First show the basic arguments for a prime still work for prime powers. (Make sure you know what the ‘basic arguments’ are!)
- (2) Second make Theorem 2 (page 42) work for general  $n$  as follows:
  - Prove the discriminant of the ring extensions  $\mathbb{Z}[\xi_{p^\alpha}]/\mathbb{Z}$  is a power of  $p$
  - Infer that the various subextensions of  $\mathbb{Q}(\xi_n)$  corresponding to the prime powers dividing  $n$  have relatively prime discriminants
  - (EXTRA CREDIT) Prove that if we have two Galois extensions  $L$  and  $L'$  of  $\mathbb{Q}$  such that  $L \cap L' = \mathbb{Q}$  with absolute discriminants  $d$  and  $d'$  which are relatively prime then the discriminant of the compositum  $L \cdot L'$  is  $d \cdot d'$ .