

W4043 HOMEWORK #4

DUE: TUESDAY, FEBRUARY 20TH, 2007

- (1) Verify that the discriminant $D(1, x, x^2)$ in the number field

$$K = \mathbb{Q}[x]/(x^3 + x - 1)$$

equals -31 directly from the definition on P 38:

$$D(x_1, \dots, x_n) = \det(\text{Tr}_{K/\mathbb{Q}}(x_i x_j)).$$

- (2) Assuming the fact that the discriminant of a number field other than \mathbb{Q} is always greater than 1 in absolute value, deduce that $\langle 1, x, x^2 \rangle$ is a basis for the ring of integers of K above and that -31 is therefore the discriminant of this number field.
- (3) Show that x generates an infinite cyclic subgroup of the group of units of the above K (i.e. of its ring of integers).