W4043 HOMEWORK #1

DUE: THURSDAY, JANUARY 25TH, 2007

(1) Let A be an integral domain. Prove that the set of units of the polynomial ring $B = A[X_1, X_2, \ldots, X_n]$ is just the set A^*.
(2) Let A be an integral domain. Prove that the set of units of the power series ring $B = A[[X_1, X_2, \ldots, X_n]]$ is the set of power series whose constant coefficient is in A^*.
(3) Let K be a field. Prove that the power series ring $K[[X]]$ is a PID by proving that any ideal is generated by X^n for some $n \in \mathbb{N}$.
(4) Find an example of a domain R where Euclid's Lemma does not hold.
(5) Give an example of a domain R and ideals I and J where $I \cdot J \neq I \cap J$.