Exercise 1. The helix \( \vec{r}_1(t) = \langle 2 \cos t, 2 \sin t, t \rangle \) intersects the curve \( \vec{r}_2(t) = \langle 2, t, t^2 \rangle \) at a unique point \( P(t_0) \).
1. Find the coordinates of this point \( P(t_0) \).
2. Find the equation of the normal plane to each curve at \( P(t_0) \).
3. What is the angle between these planes?

Exercise 2. Consider the surface \( S \) given by
\[
f(x, y) = x^3 + xy + \cos(y).
\]
1. Compute \( \frac{\partial f}{\partial x}(x, y) \) and \( \frac{\partial f}{\partial y}(x, y) \).
2. Consider the intersection of \( S \) with the plane \( y = 2\pi/3 \). We call this curve \( C \). What is the slope of the tangent line to the curve \( C \) in the plane \( y = 2\pi/3 \) at point \( (1, 2\pi/3, f(1, 2\pi/3)) \)?
3. (optional) Give the equation of this line.

Exercise 3. The velocity of a 5 kilogram ball is given by
\[
\vec{v}(t) = \langle 2 \sin t, -\cos t, 3t \rangle
\]
while the position of the ball at time \( t_0 = -1 \) is \( \langle 2, 0, 1 \rangle \).
1. Find the position of the ball at any time \( t \).
2. What is the force \( \vec{F} \) which acts on the ball at any time \( t \)?

Exercise 4 (True/False).
1. The limit \( \lim_{(x,y) \to (0,0)} \frac{x^2}{x^2+y^2+1} \) exists and equals zero.
2. The limit \( \lim_{(x,y) \to (0,0)} \frac{2xy^2}{x^2+y^2} \) exists and equals zero.
3. The limit \( \lim_{(x,y) \to (0,0)} \frac{2y^2}{x^2+y^2} \) exists and equals zero.
4. The limit \( \lim_{(x,y) \to (0,0)} \frac{\cos(x^2+y^2)-1}{x^2+y^2} \) exists and equals zero.
5. The curve \( \langle t^2 - 1, 2t^2 + 1, 3 \rangle \) describes a straight line.
6. The curves \( \langle t, t^2, t^3 \rangle \) and \( \langle (t^3 - 1), (t^3 - 1)^2, (t^3 - 1)^3 \rangle \) are the same.
7. The curves \( \langle t, t^2, t^3 \rangle \) and \( \langle (t - 1)^2, (t - 1)^3, t - 1 \rangle \) are the same.
8. For \( f(x, y) = x^2 + xy + \cos(xy) \) we have \( \frac{\partial f}{\partial x}(x, y) = x^2 + y - \sin(xy) \).
Exercise 5. We recall that for any $a \in \mathbb{R}$, we have
$$2\sin^2(a/2) = 1 - \cos(a) \text{ and } \sin(a) = 2\sin(a/2)\cos(a/2).$$

We consider the curve with equation
$$\begin{cases}
    x(t) &= t - \sin(t) \\
    y(t) &= 1 - \cos(t) \\
    z(t) &= 0
\end{cases}$$

We set $\vec{r}(t) = <x(t), y(t), z(t)>$.

1. Compute $\vec{r}'(t)$ for any parameter $t$.

2. Compute the unit tangent vector $\vec{T}(t)$ for any parameter $t \in [0, 2\pi]$. (Use the formulas given at the beginning of the exercise).

3. What is the osculating plane of the curve?

4. Compute the length of the curve between the parameters $0$ and $\pi$.

5. Compute the length of the curve between the parameters $0$ and $2\pi$.

6. Now we look at the curve in the $xy$ plane.
   
   (a) For which parameter $t$ does the curve have a vertical tangent line at the point $(x(t), y(t))$? (Use Question 1).

   (b) For which parameter $t$ does the curve have a horizontal tangent line at the point $(x(t), y(t))$? (Use Question 1).

   (c) Draw the curve in the $xy$ plane for $t \in [0, 2\pi]$. Illustrate the answer to the 2 previous questions.

7. Compute the normal vector $\vec{N}(t)$ for any parameter $t \in [0, 2\pi]$.

8. Compute the curvature $k(t)$ for any parameter $t \in [0, 2\pi]$.

9. Find the coordinates of the osculating circle for any parameter $t \in [0, 2\pi]$.

10. For which parameter $t \in [0, 2\pi]$ is the radius of the osculating circle maximal?