Exercise 1. Find the greatest common divisor \( d \) of 1109 and 4999, and then find integers \( x \) and \( y \) satisfying
\[ 1109x + 4999y = d \]

Exercise 2. Prove that if \( n \) is composite, it must have a prime factor \( \leq \sqrt{n} \).

Exercise 3. What are the last two digits of \( 3^{2011} \)?

Exercise 4. Solve the congruence
\[ x^3 + 4x + 8 = 0 \mod 15. \]

Exercise 5. Is 7 a quadratic residue mod 11?

Exercise 6. Give the list of the residues mod 12 that are invertible in \( \mathbb{Z}/12\mathbb{Z} \).

Exercise 7. Let \( p \) be a prime number.
1. For \( k \) an integer \( 1 \leq k \leq p \), prove the following equality between binomial coefficients:
\[ k \binom{p}{k} = p \binom{p - 1}{k - 1}. \]
2. For which values of \( k \) does \( p \) divide \( \binom{p}{k} \)?