Problem set 9

Due Wednesday, April 5

1. Show that every line bundle on the projective space \( \mathbb{P}^n \) is of the form \( \mathcal{O}(m) \) for some \( m \).

2. Show that \( \mathcal{O}(m) \) is naturally equivariant for the natural action of \( G = GL(n + 1) \) on \( \mathbb{P}^n \). When can \( \mathcal{O}(m) \) be made equivariant for the action of

\[
\text{Aut}(\mathbb{P}^n) = PGL(n + 1) = GL(n + 1)/\text{center}
\]

3. Compute the cohomology groups of \( \mathcal{O}(m) \) from the standard Čech complex. Identify them as representation of \( G \).

4. Construct a minimal \( G \)-equivariant resolution of the skyscraper sheaf \( \mathcal{O}_0 \) of the origin

\[
0 \in \mathbb{A}^{n+1}.
\]

Obtain a relation in \( K_G(\mathbb{P}^n) \) by restricting this resolution to \( \mathbb{A}^{n+1} \setminus \{0\} \).