Problem set 2

Due Wednesday, February 8

In this homework, we finish the classifications of discrete groups $\Gamma \subset \mathrm{Iso}(\mathbb{R}^n)$ generated by reflections of a Euclidean space $\mathbb{R}^n$. We use the notations of HW1. Concretely, our goal is to classify indecomposable matrices $A = (a_{ij})$ where

$$a_{ij} = (e_i, e_j) = -\cos(\pi/m_{ij}), \quad i \neq j,$$

corresponding to Γ as in HW1.

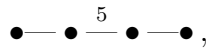
1. Suppose A and $A' \neq A$ are matrices such that

$$m_{ij} \geq m'_{ij}$$

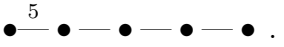
for all i and j. We write $A \succ A'$ to denote this. Use problem 7 in HW1 to show that if A corresponds to an elliptic or parabolic reflection group then A' corresponds to an elliptic reflection group.

2. Show that the elliptic and parabolic matrices listed in the wikipedia article on Coxeter groups are indeed elliptic and parabolic.

3. Show that the matrix A corresponding to the Coxeter diagram



where 5 means that $m_{23} = m_{32} = 5$, is negative definite, as is the matrix corresponding to



4. Show that for any matrix A not on Coxeter's list of elliptic and parabolic matrices there exists $A' \prec A$ that A' is either elliptic or is one one of the two matrices from Problem 3. Finish the classification.

5. Note that there is no parabolic Γ which has $m_{ij} = 5$ for some i and j. Can you explain it without referring to classification ?