The Effect of Model Risk on the Valuation of Barrier Options

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Introduction

It is known that calibrating different stochastic process models to the same vanilla option surface would yield different exotic option prices.

What is not clear is the precise magnitude of these differences within the context of models calibrated to actual market prices.

We provide a study on the effect of model risk by considering four different models: variance gamma (VG), constant elasticity of variance (CEV), local volatility, and variance gamma with stochastic arrival (VGSA).

Description of the Models

Models under consideration are

Variance Gamma (VG) Model

Constant Elasticity of Variance (CEV) Model

Local Volatility Model

Variance Gamma with Stochastic Arrival (VGSA)

Variance Gamma (VG) Process

The VG process $X(t;\sigma,\nu,\theta)$ is obtained by evaluating Brownian motion with drift θ and volatility σ at a random time given by a gamma process $\gamma(t;1,\nu)$ with mean rate unity and variance rate ν as

$$X(t;\sigma,\nu,\theta) = \theta\gamma(t;1,\nu) + \sigma W(\gamma(t;1,\nu))$$

Suppose the stock price process is given by the geometric VG law with parameters σ, ν, θ and the log price at time t is given by

$$\ln S_t = \ln S_0 + (r - q + \omega)t + X(t;\sigma,\nu,\theta)$$

where

$$\omega = \frac{1}{\nu}\ln(1 - \theta\nu - \sigma^2\nu/2)$$

is the usual Jensen's inequality correction ensuring that the mean rate of return on the asset is risk neutrally $(r - q)$.

Constant Elasticity of Variance (CEV) Model

CEV process assumes that the asset price follows the process

$$dS_t = (r - q)S_t dt + \delta S_t^{\beta+1} dW_t$$

for $t > 0$, $S_0 > 0$.

Local Volatility Model

Consider the stock price process as a solution to the stochastic differential equation

$$dS_t = (r - q)S_t dt + \sigma(S_t, t) dW(t),$$

where the function $\sigma(S, t)$ is termed the asset's local volatility function.

Variance Gamma with Stochastic Arrival (VGSA)

To obtain VGSA, we take the VG process which is a homogeneous Lévy process and build in stochastic volatility by evaluating it at a continuous time change given by the integral of a Cox, Ingersoll and Ross (CIR) process.

The mean reversion of the CIR process introduces the clustering phenomena often referred to as volatility persistence. This enables us to calibrate to market price surfaces that go across strike and maturity simultaneously.

Calibration of VG Parameters

Using out-of-the-money call and put European option prices for S&P 500 Octobet 19, 2000, we obtained following

T	σ	ν	θ	r	q	S_0
0.07934	0.2094	0.0732	-0.5045	0.0663	0.0125	1389.459
0.15585	0.2139	0.1218	-0.3710	0.0663	0.0128	1389.869
0.40504	0.1927	0.2505	-0.2859	0.0667	0.0119	1389.459
0.65424	0.1895	0.4668	-0.2156	0.0660	0.0117	1389.708
0.92273	0.1952	0.6140	-0.1994	0.0654	0.0116	1390.906

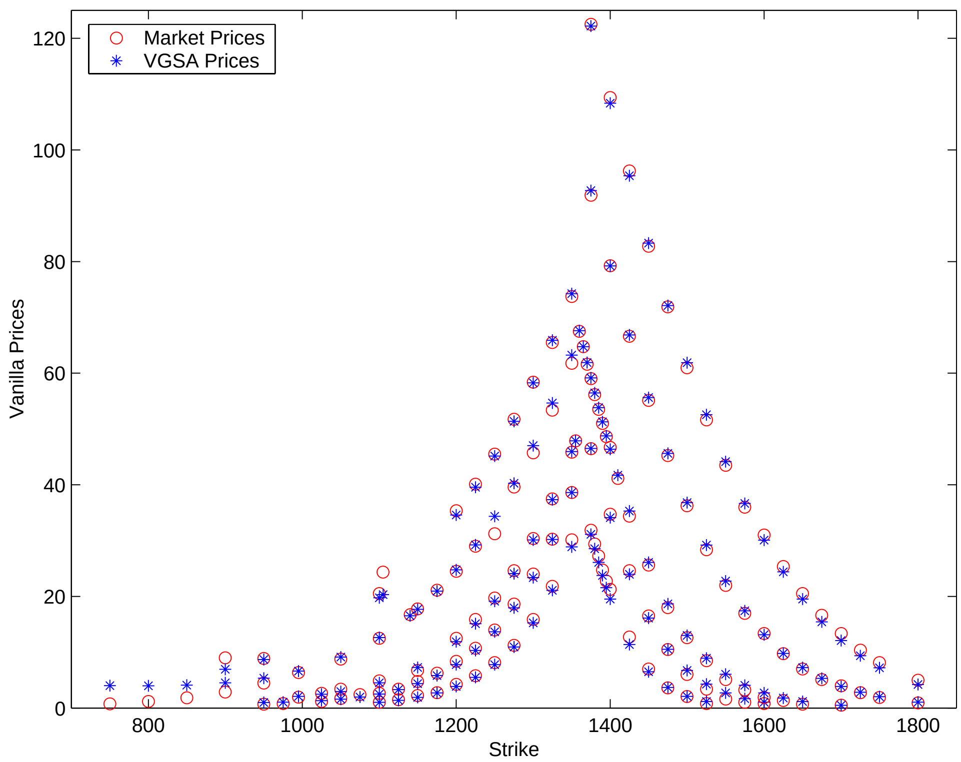
Calibration of CEV Parameters

Using out-of-the-money call and put European option prices for S&P 500 Octobet 19, 2000, we obtained following

T	σ	β	r	q	S_0
0.07934	0.21785045	-1.493591071	0.0663	0.0125	1389.459
0.15585	0.21897197	-1.494322146	0.0663	0.0128	1389.869
0.40504	0.21459526	-1.465004002	0.0667	0.0119	1389.459
0.65424	0.22271495	-2.247280943	0.0660	0.0117	1389.708
0.92273	0.22668701	-1.993795395	0.0654	0.0116	1390.906

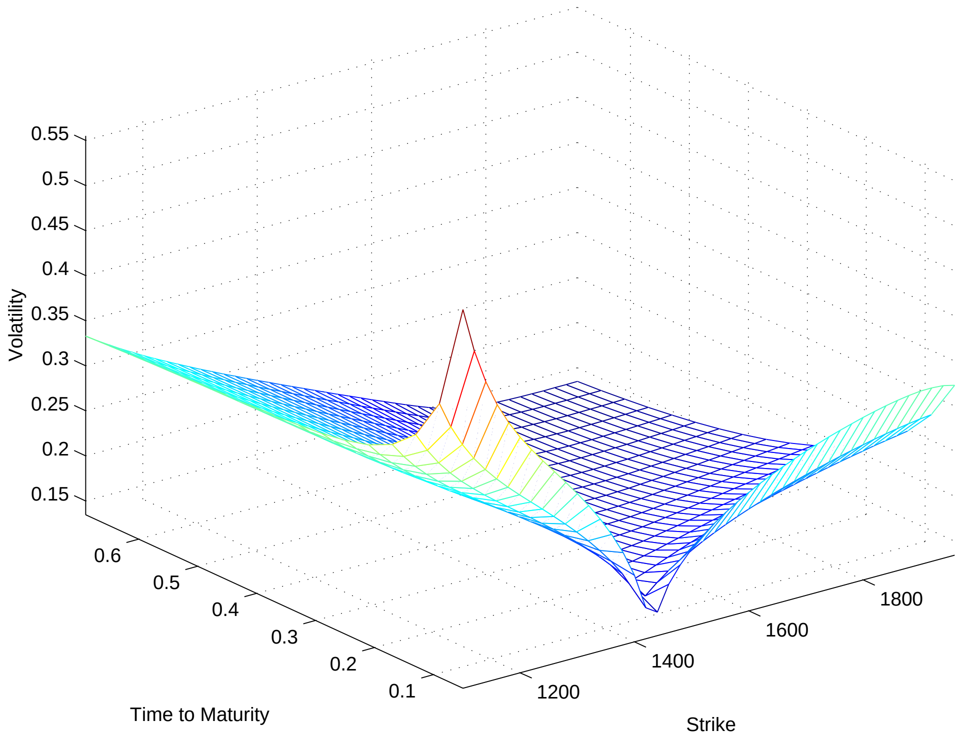
VGSA Calibration

VGSA parameters from calibration of S&P 500 prices of De-cemeber13, 2000 are as follows

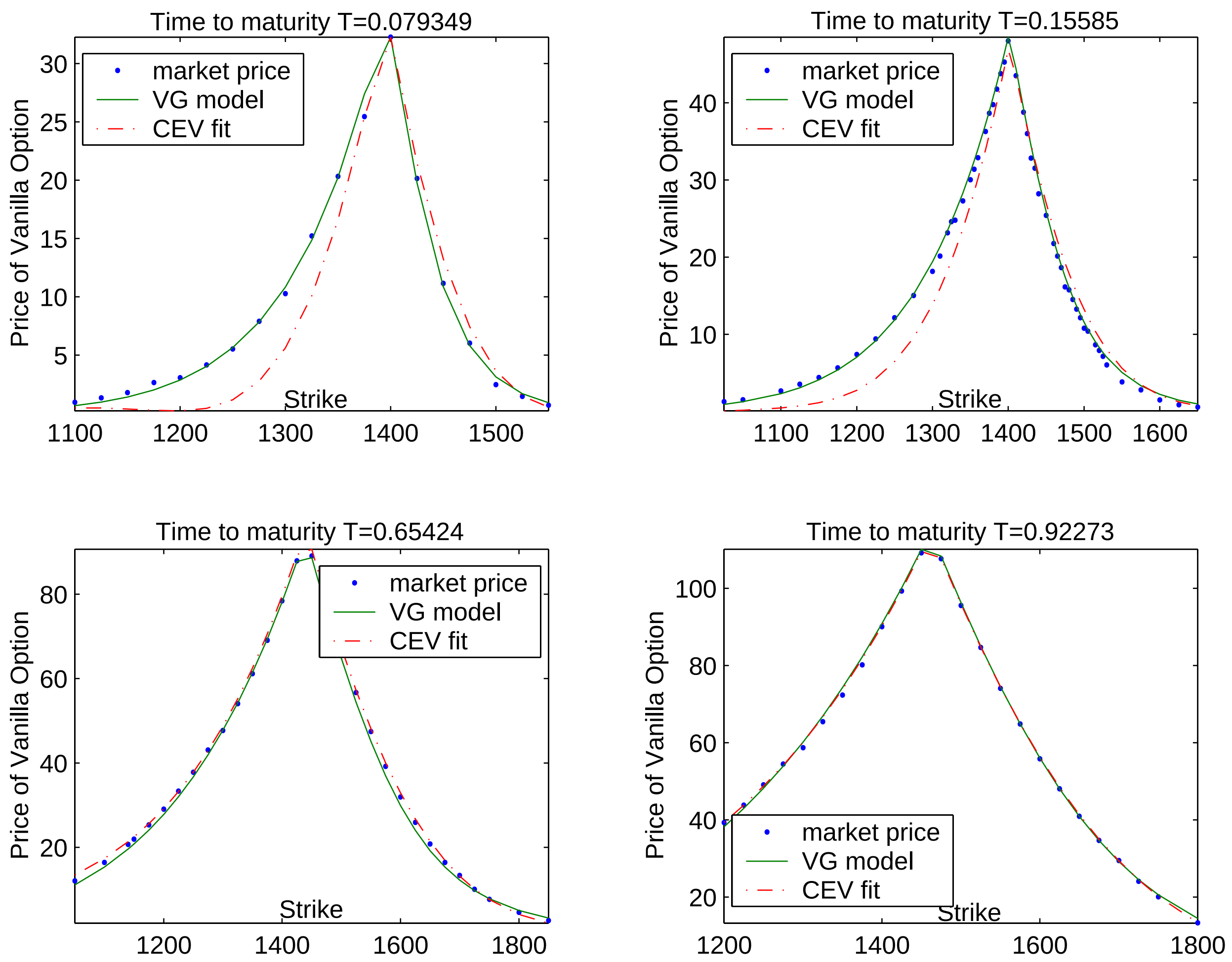


Local Volatility Calibration

This surface is obtained using the Dupire methodology applied to VGSA call prices obtained using the estimated VGSA parameters.



Assessment of the VG and CEV Fit



UOC Prices for VG Process

Applying Ito's Lemma for semi-martingle processes, one can show that $v(s,t)$ must satisfy the following partial integro-differential equation (PIDE).

$$\frac{\partial v}{\partial t} + (r - q + \omega)s\frac{\partial v}{\partial s} + \int_{-\infty}^{\infty}(v(se^y,t) - v(s,t))k(y)dy = rv$$

where $k(y)$ is the VG Lévy measure. The region in which this equation is to be solved is $\{(s,t)|0 \leq s \leq B, 0 \leq t \leq T\}$. The boundary conditions for the up-and-out call are

$$\begin{aligned} v(B,t) &= \text{Rebate} \\ v(0,t) &= 0 \text{ for } 0 \leq t \leq T \end{aligned}$$

UOC Prices for CEV, VGSA, and Local Volatility

UOC for CEV can be expressed in closed form by the eigenfunction expansion.

VGSA UOC prices are obtained via Monte-Carlo simulation.

Having derived the local volatility surface $\sigma(s,t)$ we can price the up-and-out call prices by solving the following partial differential equation

$$\frac{\partial v}{\partial t} + (r-q)s\frac{\partial v}{\partial s} + \frac{1}{2}\sigma^2(s,t)s^2\frac{\partial^2 v}{\partial s^2} = rv$$

$$\begin{aligned} v(B,t) &= \text{Rebate} \\ v(0,t) &= 0 \text{ for } 0 \le t \le T \end{aligned}$$

Numerical Results (Comparisons between VG & CEV)

Maturity		$T_1 = 0.40504$		$T_2 = 0.65424$		$T_3 = 0.92273$	
Barrier	Strike	CEV	VG	CEV	VG	CEV	VG
1600	1300	52.12	80.10	34.51	59.11	21.25	36.80
	1350	33.13	55.80	22.10	41.54	13.37	25.50
	1400	18.51	35.29	12.49	26.70	7.42	16.16
	1450	8.45	19.13	5.79	14.91	3.39	8.90
	1500	2.69	7.84	1.88	6.44	1.08	3.79
	1550	0.36	1.61	0.26	1.47	0.14	0.86
1550	1250	45.71	70.74	26.53	46.66	15.89	28.65
	1300	29.38	50.02	17.02	32.81	10.00	19.84
	1350	16.58	32.27	9.63	21.11	5.55	12.56
	1400	7.64	17.97	4.47	11.78	2.53	6.90
	1450	2.45	7.67	1.45	5.07	0.81	2.93
	1500	0.33	1.69	0.20	1.16	0.11	0.66

Cont'd

Maturity		$T_1 = 0.40504$		$T_2 = 0.65424$		$T_3 = 0.92273$	
Barrier	Strike	CEV	VG	CEV	VG	CEV	VG
1500	1200	34.05	51.61	17.57	32.49	10.40	19.97
	1250	22.03	36.59	11.26	22.81	6.54	13.80
	1300	12.51	23.66	6.37	14.63	3.63	8.71
	1350	5.80	13.24	2.96	8.15	1.65	4.78
	1400	1.86	5.68	0.96	3.50	0.53	2.03
	1450	0.25	1.26	0.13	0.79	0.07	0.45
1450	1200	12.32	20.64	5.66	12.60	3.30	7.68
	1250	7.02	13.30	3.20	8.04	1.83	4.83
	1300	3.26	7.41	1.48	4.45	0.83	2.64
	1350	1.05	3.16	0.48	1.90	0.26	1.11
	1400	0.14	0.70	0.06	0.43	0.04	0.25

Numerical Results (Comparisons between VGSA & LV)

Maturity		$T_1 = 0.25$		$T_2 = 0.50$		$T_3 = 0.75$	
Barrier	Strike	LV	VGSA	LV	VGSA	LV	VGSA
1600	1300	65.13	71.82	40.99	56.16	26.92	42.63
	1350	41.27	45.38	25.98	36.97	16.97	28.40
	1400	23.01	25.27	14.53	21.81	9.45	17.08
	1450	10.52	11.70	6.68	10.90	4.33	8.76
	1500	3.36	3.95	2.15	4.08	1.39	3.37
	1550	0.45	0.66	0.29	0.75	0.19	0.65
1550	1250	65.26	80.93	35.04	54.31	21.01	38.35
	1300	42.18	53.30	22.24	36.36	13.19	25.68
	1350	23.98	31.14	12.45	21.91	7.31	15.54
	1400	11.15	15.17	5.72	11.21	3.33	8.00
	1450	3.61	5.41	1.83	4.27	1.06	3.07
	1500	0.49	0.93	0.25	0.80	0.14	0.59

Cont'd

Maturity		$T_1 = 0.25$		$T_2 = 0.50$		$T_3 = 0.75$	
Barrier	Strike	LV	VGSA	LV	VGSA	LV	VGSA
1500	1200	56.32	79.00	26.79	46.84	14.81	31.59
	1250	36.86	53.74	17.00	31.82	9.26	21.30
	1300	21.21	32.59	9.50	19.47	5.11	12.98
	1350	9.96	16.57	4.35	10.09	2.31	6.73
	1400	3.25	6.15	1.39	3.93	0.73	2.60
	1450	0.44	1.11	0.19	0.77	0.10	0.51
1450	1200	26.00	44.17	10.92	23.62	5.49	15.24
	1250	15.04	27.47	6.09	14.57	3.02	9.31
	1300	7.09	14.32	2.78	7.66	1.36	4.84
	1350	2.32	5.45	0.88	3.02	0.43	1.88
	1400	0.31	1.01	0.12	0.59	0.06	0.37

Conclusion and Future Work

Regardless of the closeness of the vanilla fits to different models, prices of up-and-out call options (a simple case of exotic options) differ noticeably when using different stochastic processes to calibrate the vanilla options surface.

Two models, one continuous and one purely discontinuous were calibrated to single maturities: (VG and CEV models.) It was observed that for reasonable levels of the spot price the VG model had a substantially higher price for the up and out call option.

A similar conclusion was reached when comparing the pure jump $VGSA$ model calibrated to the surface when it is compared to its continuous local volatility counterpart.

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How such assets should be priced and what are the appropriate prices to quote for path dependent options?

It is clear from this investigation that even simple exotic options are far from the span of vanilla options trading at a single date.

The difference in pricing across models will be explained if we answer the question of what is to be done with the money obtained from the sale.

This money is to be transferred into a trading strategy that results in a hedged P&L deemed an acceptable risk (on an expost basis.)

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This paper serves to document and highlight the substantial issues and open questions in the field of barrier option pricing. No doubt future research will provide greater guidance and resolution of the problems presented here.