MATH V1201 SECTIONS 002 & 003 HOMEWORK 8
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This is version 2, with a correction to Problem (IV.1).

1. Some Stewart problems

(I.1) Stewart 14.4.6.
(I.2) Stewart 14.4.19.
(I.3) Stewart 14.4.46.
(I.4) Stewart 14.5.28, but do *not* use Equation 6: just apply the chain rule to the equation $\cos(xy) - 1 - \sin(y) = 0$. (You may use equation 6 to check your answer, if you like.)
(I.5) Stewart 14.5.34, but do *not* use Equation 7: just apply the chain rule to the equation $yz + x\ln(y) - z^2 = 0$. (You may use equation 7 to check your answer, if you like.)
(I.6) Stewart 14.5.42.
(I.7) Stewart 14.5.51.

2. Chain rule and change of coordinate systems

(II.1) Consider the parametric curve in polar coordinates $(r, \theta) = (1 + \sin(t), t)$.
  (a) Use the chain rule to find an expression for the speed of the curve at time t.
  (b) Find the arc length of the curve between $t = 0$ and $t = 2\pi$.
(II.2) Consider a parametric curve in spherical coordinates $\vec{r} = (\rho(t), \theta(t), \phi(t))$. Use the chain rule to find an expression for the speed of the curve $\vec{r}$. (Hint: imitate what we did in class for polar coordinates.)

3. Implicit differentiation

(III.1) Suppose x and y satisfy the relationship $ex^2y^3 = e^{xy}$. Use implicit differentiation to compute $y'(x)$ and $x'(y)$ at $(1, 1)$.
(III.2) Suppose x, y, and z satisfy $\sin(xyz) = xy + yz$. Use implicit differentiation to compute $\frac{\partial z}{\partial x}(1, 1, \pi)$ and $\frac{\partial z}{\partial y}(1, 1, \pi)$.
(III.3) Consider the curve C defined by $\cos(xy) - (x + y)/(2\sqrt{\pi}) = 0$ in $\mathbb{R}^2$.
- Notice that $(-\sqrt{\pi}, -\sqrt{\pi})$ lies on the curve. Using problem (IV.1), explain why the curve C implicitly defines y as a function of x near $(-\sqrt{\pi}, -\sqrt{\pi})$.
- Compute $y'(x)$ at the point $(-\sqrt{\pi}, -\sqrt{\pi})$.
- The point $(2\sqrt{\pi}, 0)$ also lies on the curve C. Does C define y implicitly as a function of x near $(2\sqrt{\pi}, 0)$? Does C define x implicitly as a function of y near $(2\sqrt{\pi}, 0)$? Explain.
- What happens when you try to compute $y'(x)$ at $(2\sqrt{\pi}, 0)$ using implicit differentiation?

(III.4) Consider the ellipsoid E given by $x^2/4 + y^2/9 + z^2 = 3$.
- At the point $(2, 3, 1)$, E defines z implicitly as a function of x and y. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
- At what points on E does E *not* define z implicitly as a function of x and y?

4. Mathematica

(IV.1) (This goes along with problem (III.3).) Use `ContourPlot` to plot the curve $\cos(xy) - \frac{x+y}{2\sqrt{\pi}} = 0$ with plot region $-6 \leq x \leq 6$, $-6 \leq y \leq 6$. Plot it again with plot region $-2.25 \leq x \leq -1$, $-2.25 \leq y \leq -1$.

(IV.2) Suppose we wanted to define $f(u, v) = u^2 + v^2$. In Mathematica, you do this like:

```
f[u_,v_] := u^2+v^2
```

(Try it.)

(IV.3) Then you can compute $f(1, 2)$ just as you would expect:

```
f[1,2]
```

(Try it.)

(IV.4) Now, we can chain functions together. To define $u(t) = e^t$ use:

```
u[t_] := Exp[t]
```

(Try it.)

(IV.5) Define $v(t) = \sin(t)$.

(IV.6) We can now compose functions: $f(u(t), v(t))$ is

```
f[u[t],v[t]]
```

(Try it.)

(IV.7) Derivatives of compositions work exactly as you would expect; for instance:

```
D[f[u[t],v[t]],t]
```

(Try it.)

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