MATH W4051 PROBLEM SET 8
DUE OCTOBER 29, 2008.

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(1) (From Hatcher): Show that composition of paths has the following cancellation property: Let γ_0, γ_1 be paths from p to q and η_0, η_1 paths from q to r. Suppose that $\gamma_0 * \eta_0 \sim \gamma_1 * \eta_1$ (rel endpoints) and $\eta_0 \sim \eta_1$ (rel endpoints). Then $\gamma_0 \sim \gamma_1$ (rel endpoints).

(2) Munkres 52.1

(3) Munkres 55.2

(4) Munkres 55.4 parts (a)–(d)

(5) Does every continuous map $S^2 \to S^2$ have a fixed point? If so, prove it. If not, give a counterexample, and see if you can find a more restrictive statement which you think is true.

(6) Let X and Y be path-connected spaces.
   (a) Prove that $\pi_1(X \times Y) = \pi_1(X) \times \pi_1(Y)$.
   (b) Conclude that T^2 is not homeomorphic to S^2.
   (c) What is $\pi_1(\mathbb{R}^2 \setminus 0)$?

(7) (From Hatcher): Define $f\colon S^1 \times [0,1] \to S^1 \times [0,1]$ by $f(\theta, s) = (\theta + 2\pi s, s)$. So, f restricts to the identity map on the two boundary circles of $S^1 \times [0,1]$.
   (a) Show that f is homotopic to the identity map by a homotopy fixing one of the two boundary circles (i.e., rel $S^1 \times \{0\}$).
   (b) Show that f is *not* homotopy to the identity map by a homotopy fixing both boundary circles (i.e., rel $S^1 \times \{0,1\}$).
   Hint: Consider what f does to the path $s \mapsto (\theta_0, s)$ for some $\theta_0 \in S^1$.

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