

**BONUS PROBLEM FROM MATH W4051 FINAL
EXAM
DECEMBER 16, 2008**

INSTRUCTOR: ROBERT LIPSHITZ

Consider $UTS^2 = \{(x, v) \in S^2 \times \mathbb{R}^3 \mid |v| = 1, v \cdot x = 0\}$. (Here, the dot product comes from viewing S^2 as sitting inside \mathbb{R}^3 . This is the “unit tangent space to S^2 .”)

- (1) Prove that UTS^2 is path connected.
- (2) Compute $\pi_1(UTS^2)$.
- (3) Conclude that S^2 does not admit a nonvanishing tangent vector field. (Hint: show that if it did then $UTS^2 \cong S^2 \times S^1$.)

E-mail address: r12327@columbia.edu