## MATH W4052 PROBLEM SET 8 DUE APRIL 4, 2011.

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- (1) Lickorish, Exercise 6.3.
- (2) Let M be a finitely presented module over a ring R. Prove that the  $r^{\text{th}}$  elementary ideal of M,  $\mathcal{E}_r$ , is independent of the choice of presentation matrix used to define it.
- (3) Let  $\Gamma_1$  and  $\Gamma_2$  be embedded graphs in  $\mathbb{R}^3$ . Suppose that there is a ball B in  $\mathbb{R}^3$  so that  $\Gamma_1 \cap (\mathbb{R}^3 \setminus B) = \Gamma_2 \cap (\mathbb{R}^3 \setminus B)$ , and that  $\Gamma_1 \cap B$  and  $\Gamma_2 \cap B$  consist of two arcs each, and look like this (in the style of knot diagrams):



Prove: for any such  $\Gamma_1, \Gamma_2, H_1(\mathbb{R}^3 \setminus \Gamma_1) \cong H_1(\mathbb{R}^3 \setminus \Gamma_2)$ . Is the same true with  $H_1$  replaced by  $\pi_1$ ?

(4) In class, we considered the following covering space of the figure 8:



What subgroup of  $\pi_1$  of the figure 8 does this correspond to? Show that this subgroup is not normal. (This may take some work.)

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