MATH W4052 PROBLEM SET 6
DUE FEBRUARY 28, 2011.

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(1) Compute a Seifert matrix and the signature and determinant of:
   (a) The trefoil knot.
   (b) The figure 8 knot.
   (c) The knot 5_2.
   You're welcome to use computer software to compute the determinants and signatures of the relevant matrices, if you like, and to check your answers against the table in the book.
(2) Cromwell 6.9.11.
(3) Cromwell 6.9.7.
(4) Recall the definition of n-colorability from Homework 2.
   (a) Explain how n-colorability is equivalent to the system of equations $A\vec{v} = 0$ having a 2-dimensional (or higher) solution space, where A is a particular $c \times c$ matrix over $\mathbb{Z}/n$.
   (b) Explain why the sum of the elements in each row of A is zero, and the sum of the elements in each column of A is zero. (Hint: look at an example or two.)
   (c) Using the previous part, explain why n-colorability is equivalent to another matrix A' having determinant 0 modulo n.
   (d) Write A' for (some projection of) the trefoil, figure 8 knot and 5_2. Compute its determinant. For which n are these knots n-colorable?
   (e) Formulate a conjecture.
(5) Read the rest of the exercises in Chapter 6 of Cromwell. (No need to write anything.)

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