MATH G4307 PROBLEM SET 7
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The problem set looks long, but most of the problems are quite easy / short. Problems (E8)–(E12) are based on material we will cover after the midterm.

Exercises to turn in:

(E1) Prove: if A, B and C are abelian groups such that there is a short exact sequence $0 \to A \to B \to C \to 0$ then $\operatorname{rank}(B) = \operatorname{rank}(A) + \operatorname{rank}(C)$. (We used this in class.)
(E2) Hatcher 2.2.22 (p. 157). (This is essentially the same as an optional problem from last week.) In the problem, you may replace "CW complex" by "simplicial complex" if you prefer.
(E3) Hatcher 2.2.23 (p. 157).
(E4) Hatcher 2.B.2 (p. 176).
(E5) Hatcher 2.C.2.
(E6) Hatcher 2.C.4.
(E7) Give a map $f\colon S^2 \to S^2$ such that f is homotopic to the identity and f has exactly one fixed point. How does this square with Problem (E6)?
(E8) Hatcher 2.2.7 (p. 155).
(E9) Hatcher 2.2.8 (p. 155).
(E10) Hatcher 2.2.14 (p. 156).
(E11) Hatcher 2.2.15 (p. 156).
(E12) Hatcher 2.2.19 (p. 157).

Problems to think about but not turn in:

(P1) Let R be a ring. Consider the free abelian group G generated by the set of finitely-generated R-modules. Define $K_0(R)$ to be the quotient of G by the relations that $N = M + P$ whenever there is a short exact sequence

$$0 \to M \to N \to P \to 0.$$

Prove that $K_0(\mathbb{Z})$ is isomorphic to $\mathbb{Z}$, via the map $M \mapsto \operatorname{rank}(M)$.
(P2) Read through the remaining problems in sections 2.2, 2.B and 2.C, and do any that seem difficult, surprising or interesting.

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