

MATH G4307 PROBLEM SET 1
DUE SEPTEMBER 13, 2011.

INSTRUCTOR: ROBERT LIPSHITZ

Exercises to turn in:

- (E1) Hatcher Exercise 0.6 (p. 18).
- (E2) Hatcher Exercise 0.10 (p. 19).
- (E3) Hatcher Exercise 0.11 (p. 19).
- (E4) Hatcher Exercise 0.16 (p. 19).
- (E5) Hatcher Exercise 0.20 (p. 19).
- (E6) Recall that the *disjoint union topology* has the following property: given spaces X, Y, Z and continuous maps $f: X \rightarrow Z, g: Y \rightarrow Z$ there is a unique continuous map $h: X \amalg Y \rightarrow Z$ so that the following diagram commutes:

$$\begin{array}{ccc} X \amalg Y & \longleftarrow & Y \\ \uparrow & \searrow h & \downarrow g \\ X & \xrightarrow{f} & Z \end{array}$$

Here, the maps $X \rightarrow X \amalg Y$ and $Y \rightarrow X \amalg Y$ are the canonical inclusions. Moreover, this property characterizes the disjoint union topology. (If you are not familiar with this property, prove it.)

What operation has the analogous property for *based spaces*? (Hint: we saw it in class.) Formulate the property precisely and prove your answer satisfies it.

Problems to think about but not turn in:

- (P1) With Exercise (E6) in mind, what are analogues $X \vee Y$ for other classes of mathematical objects (e.g., sets, groups, abelian groups, rings)? Are there (well-known) classes of objects for which \wedge does not exist?
- (P2) Read through the remaining problems in Chapter 0, and do any that seem difficult or surprising.
- (P3) Remember that you are expected to read the material we will cover in class *before class*. For next week, this is the first section of Chapter 1 (up to the first set of exercises).

E-mail address: lipshitz@math.columbia.edu