MATH W4051 PROBLEM SET 8 PART 1 OF 2
DUE NOVEMBER 17, 2009.

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For reasons of "I'm not completely sure what we'll cover next week yet, but I want you to get a feeling for homotopy equivalence now," the homework is coming in two parts. Note that you have a week and a half to do it.

(1) Let $f, g\colon S^1 \to S^1$ be continuous maps and $x_0 \in S^1$.
   (a) Assume that $f(x_0) = g(x_0)$ and f is homotopic to g. Prove that $f_* = g_*\colon \pi_1(S^1, x_0) \to \pi_1(S^1, f(x_0))$. Conclude that $\deg(f) = \deg(g)$.
   (Hint: I did not say f is homotopic to g rel x_0; this is the tricky part. Use Munkres Theorem 58.4.)
   (b) Without the assumption that $f(x_0) = g(x_0)$ prove that $\deg(f) = \deg(g)$.
   (c) Prove that the degree of f is independent of the choice of x_0.

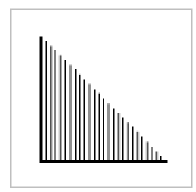
(2) Munkres 58.2

(3) Munkres 58.4

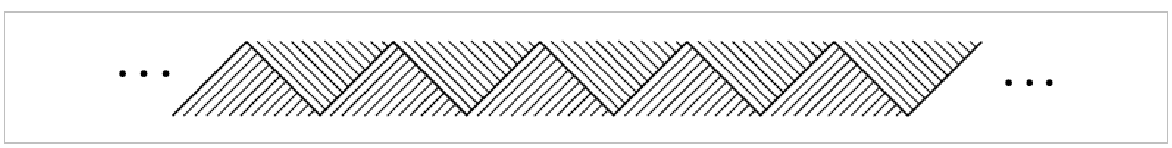
(4) Munkres 58.6

Optional: Hatcher, exercise 6(a,b), p. 18, quoted here:

6. (a) Let X be the subspace of $\mathbb{R}^2$ consisting of the horizontal segment $[0,1]\times\{0\}$ together with all the vertical segments $\{r\}\times[0,1-r]$ for r a rational number in $[0,1]$. Show that X deformation retracts to any point in the segment $[0,1]\times\{0\}$, but not to any other point. [See the preceding problem.]



(b) Let Y be the subspace of $\mathbb{R}^2$ that is the union of an infinite number of copies of X arranged as in the figure below. Show that Y is contractible but does not deformation retract onto any point.



If you want more practice, here are some suggested problems in Hatcher. All from Chapter 0 (p. 18): 1, 2, 4, 5, 10, 12, 16, 17 (repace "2-dimensional cell complex" with "space"), 20.

(Hatcher's *Algebraic Topology* book is available at: `http://www.math.cornell.edu/~hatcher/`)

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