MATH V1201 PROBLEM SET 3
DUE SEPTEMBER 29, 2009.

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(1) In the textbook:
(§12.5) 37, 56, 60, 66, 76.
(§12.6) 10, 21–28 (they're short), 46, 49.

(2) Any three points in $\mathbb{R}^3$ lie on a plane, but four randomly-chosen points won't. Here's a way of testing if four points $p_1, p_2, p_3, p_4 \in \mathbb{R}^3$ are co-planar. Let $\vec{v}_1 = p_2 - p_1$ be the vector from p_1 to p_2, $\vec{v}_2$ the vector from p_1 to p_3 and $\vec{v}_3$ the vector from p_1 to p_4. Then p_1, p_2, p_3, p_4 are coplanar if and only if $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) = 0$.
   (a) Use the test to determine if the points $(1, 0, 1)$, $(2, 1, 0)$, $(0, 2, 0)$ and $(1, 1, 1)$ are coplanar?
   (b) Use the test to determine if the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ and $(2/3, 1/6, 1/6)$ are coplanar?
   (c) Explain briefly why this test for coplanarity works. (There are lots of possibly explanations, none of which should take more than a few sentences.)
   Suggestion. Explain this out loud to one of your friends or fellow classmates before writing it up.

(3) Consider the function $f(x, y, z) = x^2 + y^2 + z^2 - (x^2 + y^2 + z^2)^3$.
   (a) What do the level sets $f(x, y, z) = c$ look like for $c = 0$? $c = -1$? $c = 3$? (You're welcome to use a computer if it helps. Or, doing (3b) first might help.)
   (b) Write f as a function of spherical coordinates, $f(\rho, \theta, \phi)$.
   (c) Find the maximum value of f. (Hint: use (3b) and one-variable calculus.)

If you had trouble with	Do problems
12.5.37	12.5.33–12.5.38
12.5.56	12.5.53–12.5.53, 12.5.57, 12.5.58
12.5.60	12.5.59
12.5.66	12.5.19–12.5.22, 12.5.65
12.5.76	12.5.75
12.6.10	12.6.9
12.6.21–28	12.6.3–12.6.8, 12.6.11–12.6.20. And play with plugging some equations into a computer
12.6.46	12.6.45
12.6.49	12.6.50?

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