CALCULUS III: HOMEWORK 2

This assignment is due on Thursday, July 17th.

There is an additional component to the homework which you must access through WebAssign. If you are auditing the class and so have not purchased WebAssign, but would like additional problems to help with following the material, please contact me by email.

(1) Do the following exercises from the book: 12.5, Exercise 77, 13.2 Exercise 53, 13.3 Exercise 57, 13.3 Exercise 58.

(2) This problem is get some more familiarity with linear algebra, using some of the operations and geometric constructions we talked about on Thursday.

(a) Fix a vector \( \mathbf{a} = (a_1, \ldots, a_n) \in \mathbb{R}^n \). Consider the following function
\[
L : \mathbb{R}^n \to \mathbb{R}
\]
\[
\mathbf{v} \mapsto \mathbf{a} \cdot \mathbf{v}
\]
Show that this map is linear. By the theorem shown in class, this means that \( L \) corresponds to a matrix \( A \), i.e. there is a matrix \( A \) so that
\[
L(\mathbf{v}) = A\mathbf{v}
\]
for all \( \mathbf{v} \). What are the dimensions of \( A \)? Find \( A \).

(b) Fix a vector \( \mathbf{a} = (a_1, a_2, a_3) \in \mathbb{R}^3 \). Consider the following function
\[
L : \mathbb{R}^3 \to \mathbb{R}^3
\]
\[
\mathbf{v} \mapsto \mathbf{a} \times \mathbf{v}
\]
Show that this map is linear. By the theorem shown in class, this means that \( L \) corresponds to a matrix \( A \), i.e. there is a matrix \( A \) so that
\[
L(\mathbf{v}) = A\mathbf{v}
\]
for all \( \mathbf{v} \). What are the dimensions of \( A \)? Find \( A \).

(c) Fix a vector \( \mathbf{a} = (a_1, \ldots, a_n) \in \mathbb{R}^n \). Consider the following function
\[
L : \mathbb{R}^n \to \mathbb{R}^n
\]
\[
\mathbf{v} \mapsto \text{proj}_\mathbf{a}\mathbf{v}
\]
Show that this map is linear. By the theorem shown in class, this means that \( L \) corresponds to a matrix \( A \), i.e. there is a matrix \( A \) so that
\[
L(\mathbf{v}) = A\mathbf{v}
\]
for all \( \mathbf{v} \). What are the dimensions of \( A \)? Find \( A \).