CALCULUS III: HOMEWORK 1

This assignment is due on Monday, July 14th.

There is an additional component to the homework which you must access through WebAssign. If you are auditing the class and so have not purchased WebAssign, but would like additional problems to help with following the material, please contact me by email.

(1) Do the following exercises from the book: 12.2, Exercise 51; 12.5, Exercise 75.

(2) This is an amusing computation in $n$-dimensions.

(a) We start with $n = 2$. Consider the unit square in the plane centered at the origin, with vertices at $(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, -\frac{1}{2}), (-\frac{1}{2}, -\frac{1}{2})$, and $(-\frac{1}{2}, \frac{1}{2})$. Draw this square. Now, place 2-dimensional balls $B_1, B_2, B_3,$ and $B_4$ all of the same radius, centered at each of these vertices and with radius $r$ just large enough that each of the four circles is mutually tangent to each of its two neighbors. What is this radius $r$? Draw in all of the $B_i$s. Finally, place a ball $B_{central}$ centered at the origin or radius $R$ just large enough so that it is mutually tangent to $B_1, B_2, B_3,$ and $B_4$. What is $R$? Draw in $B_{central}$. Note that $B_{central}$ lies inside the square.

(b) We do $n = 3$. We start with the unit cube centered at the origin, with eight vertices with coordinates $(\pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2})$. Draw the cube. Place 3-dimensional balls $B_1, ..., B_8$ all of the same radius $r$ centered at each of these vertices so that each of the eight spheres is mutually tangent to its three neighbors. Draw in the spheres $B_i$. Finally, place a ball $B_{central}$ centered at the origin of radius $R$ just large enough so that it is mutually tangent to all of the $B_1, ..., B_8$. What is $R$? Draw in $B_{central}$. Note that $B_{central}$ lies inside the cube.

(c) Now for general $n$. We start with the unit hypercube centered at the origin. How many vertices does it have, and where are they located? Place $n$-dimensional balls $B_i$ of radius $r$ at each of the vertices so that each of the balls are mutually tangent to its $n$ neighbors. Finally, place a ball $B_{central}$ centered at the origin of radius $R = R(n)$ (this notation is to emphasize that this radius depends on the dimension $n$) just large enough so that it is mutually tangent to each of the $B_i$. What is $R(n)$? Plug in any $n \geq 5$. If you have done the problem correctly, you should be confused.