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# REMARK ON THE PAPER "ACTIONS OF FINITE CYCLIC GROUPS ON QUASICOMPLEX MANIFOLDS"

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In the author's paper [1] two theorems were formulated (Theorems 1.11 and 2.2), asserting that certain conditions on a collection of $Z_m$-bundles were necessary and sufficient for this collection to be the collection of normal bundles to the fixed submanifolds of a unitary $Z_m$-manifold. Unfortunately, these conditions are in fact only sufficient. However, the methods of [1] allow one, at the cost of purely technical complications, to obtain necessary and sufficient conditions.

We consider the category whose objects are collections $(X, \{\xi_i\})$, where $X$ is a $G$-manifold and $\{\xi_i\}$ is a finite collection of $G$-bundles over $X$. The bordism groups $U^G_{n,\mu}$ in this category ($n = \dim_R X$; $\mu = (\mu_1, \ldots)$ is a "multi-index", $\mu_i = \dim_{\mathbf{C}} \xi_i$) replace for us the bordism groups of $G$-manifolds, which, incidentally, are the special case of $U^G_{n,\mu}$ for $\mu_i \equiv 0$.

For a collection of $Z_{p^k}$-bundles $\{\xi_i\}$ over $X$ ($p$ prime), let $\zeta_{is}$ be the restriction of $\xi_i$ to a fixed submanifold $F_s$ of $X$, and $\nu_s$ the normal bundle of $F_s$ in $X$. As usual, the collection of $Z_{p^k}$-bundles $\nu_s$, $\{\zeta_{is}\}$ over the trivial $Z_{p^k}$-manifolds $F_s$ defines a homomorphism

$$\beta^k : U^{Z_{p^k}}_{n,\mu} \to R^{Z_{p^k}}_{n,\mu} = \sum U_{2l}\left(\prod_{j=1}^{p^k-1} BU(n_j) \times \prod_{l}^{0\leqslant j\leqslant p^k-1} BU(n_{l,j})\right),$$

where the sum is taken over those collections of nonnegative integers $(l, \{n_j\}, \{n_{i,j}\})$ for which $2(\Sigma_j n_j + l) = n$ and $\Sigma_j n_{i,j} = \mu_i$.

We denote by $\Psi$: $R^{Z_{p^k}}_{*,*} \to R^{Z_{p^k}}_{*,*}$ the homomorphism induced by the change of indices $(i, j) \to (pi + s, j')$ $(0 \le s \le p-1$, $j \equiv s \,(\mathrm{mod}\ p)$, $j' = (j-s)/p)$, and also $j \to (s, j')$.

For $\omega = (i_1, \ldots, i_n)$, let $v_\omega(u_1, \ldots, u_n)$ be the series obtained by symmetrization of the series $\Sigma_{s=1}^n u_s^{i_s} [CP(u_s)]^{-1}$, where $CP(u) = \Sigma_0^\infty [CP^m] u^m$. For each collection $\omega_i$ of length $\mu_i$ there is defined a homomorphism $V_{\omega_i}$, whose value on the additive generator $[M] \times \Pi_s (CP^{m_s}_{i,j_s})$ of the group $U_{2N}(\Pi_{j=0}^{p^k-1} BU(n_{i,j}))$ is equal to $[u]^N_{p^k-1} \times [M] \times$ {the series obtained from $v_{\omega_i}(\ldots, f([u]_{j_s}, v_s), \ldots)$ by replacing $v_s^k$ by $[CP^{m_s-k}]$}.

We define homomorphisms $D\alpha_j$ as follows: if $(j, p) = 1$, then

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$$D\alpha_j\left([M]\times\prod_{s=1}^{n_j}(CP_j^{m_s})\right)=[u]_{p^{k-1}}^m\cdot[M]\cdot\prod_{s=1}^{n_j}\left(\frac{u}{[u]_j}\right)^{m_s+1}B_{m_s}([u]_j),$$

where $\dim_C[M]=m$; but if $p$ divides $j$, then

$$D\alpha_j\left([M]\times\prod_{s=1}^{n_j}(CP_j^{m_s})\right)=[u]_{p^{k-1}}^m\cdot[M]\cdot\prod_{s=1}^{n_j}\left(\frac{[u]_{p^{k-1}}}{[u]_j}\right)^{m_s+1}B_{m_s}([u]_j).$$

We denote by $[D\alpha]_{\vec{\omega}}$, $\vec{\omega}=(\omega_1,\ldots)$, the tensor product of the homomorphisms $D\alpha_j$ and $V_{\omega_i}$,

$$[D\alpha]_{\vec{\omega}}: R_{2n,\mu}^{Z_{p^k}}\to U^*[[u]]/\theta_p([u]_{p^{k-1}})=0.$$

We introduce an additional graduation in $R_{*,*}^{Z_{p^k}}$, by making each collection $(\{n_j\},\{n_{i,j}\})$ correspond to the number $d=\sum_{(j,p)=1}n_j$. Now let $\rho_d$ be the "homogeneous component" of $\rho\in R_{*,*}^{Z_{p^k}}$.

**Theorem.** *A bordism class* $\rho\in R_{2n,\mu}^{Z_{p^k}}$ *belongs to* Im $\beta^k$ *if and only if* $\Psi(\rho)\in$ Im $\beta^{k-1}$ *and for any* $\vec{\omega}$ *the quantity*

$$\sum_{d=0}^{n}\left(\frac{u}{[u]_{p^{k-1}}}\right)^{n-d}[D\alpha]_{\vec{\omega}}(\rho_d)$$

*is divisible by* $u^n$ *in the ring* $U^*[[u]]/\theta_p([u]_{p^{k-1}})=0$.

To describe Im $\beta^{Z_m}$ in the case when $m$ is divisible by at least two primes, it is necessary to insert an analogous correction into the hypothesis of Theorem 2.2 of [1].

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BIBLIOGRAPHY

1. I. M. Kričever, *Actions of finite cyclic groups on quasicomplex manifolds*, Mat. Sb. **90** (**132**) (1973), 306–319 = Math. USSR Sb. **19** (1973), 305–319.

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