ALGEBRAIC CURVES W4045

1. Homework, week 4, due October 6

1. (a) Show that if $\alpha \in \mathbb{C}^{\times}$, then for any lattice $\Lambda \subset \mathbb{C}$, there is a holomorphic isomorphism between the complex tori $\mathbb{C}/\Lambda$ and $\mathbb{C}/\alpha\Lambda$. Show that we may therefore assume that any complex elliptic curve is isomorphic to $\mathbb{C}/\Lambda$ where Λ is of the form $\mathbb{Z} \oplus \mathbb{Z} \cdot \tau$ for some $\tau = x + iy$ with $y > 0$.

(b) When does a complex elliptic curve have an automorphism other than the identity and $z \mapsto -z$? What are the possible orders of automorphisms?

2. Exercises 2.4, 2.8, 2.9 and 2.10 in the Reid book.

3. In the last line of the proof of Pascal's Theorem, it is claimed that, if Γ is a line pair then some of the six lines of the diagram must coincide. Prove this by counting intersections of lines.