ALGEBRAIC CURVES W4045

1. Homework, week 2, due September 22

In what follows, k is a field. If necessary, you may assume k is algebraically closed.

1. Consider the parametrization of the closure in $\mathbb{P}^2(k)$ of the line $y = mx + b$:

$$\mathbb{P}^1(k) \ni (S : T) \mapsto (S : mS + bT : T) \in \mathbb{P}^2(k).$$

(a) Show that this defines a bijection between $\mathbb{P}^1(k)$ and its image in $\mathbb{P}^2(k)$.

(b) Show that every line in $\mathbb{P}^2(k)$ is of the form $V(aX + bY + cZ)$ for a triple $(a, b, c) \in k^3$. Show that this defines a bijection between $\mathbb{P}^2(k)$ and the set of lines in $\mathbb{P}^2(k)$ via

$$(a : b : c) \mapsto V(aX + bY + cZ).$$

2. Consider the (Babylonian?) parametrization of the conic $V(X^2+Y^2-Z^2) \subset \mathbb{P}^2(k)$ by $(S : T) \in \mathbb{P}^1(k)$:

$$X = 2TS;\ Y = T^2 - S^2;\ Z = T^2 + S^2.$$

Is this a bijection? Justify your answer.

3. Exercises 1.5, 1.6, 1.8, 1.9 of Miles Reid's book.

4. (Optional) Exercise 1.7 of Reid's book.