ALGEBRAIC NUMBER THEORY W4043

Homework, week 6, due October 22

1. (a) Let $q(X,Y) = aX^2 + bXY + cY^2$ be a positive-definite binary quadratic form with integer coefficients. Assume it has discriminant $\Delta = -20$ and is reduced. Recall that a reduced quadratic form has the property that $a \leq \sqrt{|\Delta|/3} \approx 2.581$. Give the possible values for (a, b, c).

(b) Show that every ideal class in $K = \mathbb{Q}(\sqrt{-5})$ has an ideal of norm at most 2. You can use the estimates $2/\pi \approx .637$ and $\sqrt{5} \approx 2.24$.

(c) Find a prime ideal I in $\mathcal{O}_K$ whose norm form $q_I(x) = N_{K/\mathbb{Q}}(x)/N(I)$ is a reduced quadratic form with discriminant -20 and distinct from the form $X^2 + 5Y^2$.

2. Assignments from Hindry's book: Exercises 6.18 and 6.20, pp. 120-121.

You can skip the last sentence of 6.18, starting "Conclude that ..."; it goes beyond what has been covered in class.

Bonus: Exercise 6.19 (you should try it in any case but don't feel obliged to hand it in).