1. (a) Let \( q(X, Y) = aX^2 + bXY + cY^2 \) be a positive-definite binary quadratic form with integer coefficients. Assume it has discriminant \( \Delta = -20 \) and is reduced. Recall that a reduced quadratic form has the property that \( a \leq \sqrt{|\Delta|/3} \approx 2.581 \). Give the possible values for \((a, b, c)\).

(b) Show that every ideal class in \( K = \mathbb{Q}(\sqrt{-5}) \) has an ideal of norm at most 2. You can use the estimates \( 2/\pi \approx .637 \) and \( \sqrt{5} \approx 2.24 \).

(c) Find a prime ideal \( I \) in \( \mathcal{O}_K \) whose norm form \( q_I(x) = N_{K/\mathbb{Q}}(x)/N(I) \) is a reduced quadratic form with discriminant \(-20\) and distinct from the form \( X^2 + 5Y^2 \).


You can skip the last sentence of 6.18, starting ”Conclude that ...”; it goes beyond what has been covered in class.

**Bonus:** Exercise 6.19 (you should try it in any case but don’t feel obliged to hand it in).