REPRESENTATION THEORY W4044

Homework, week 8, due April 1

1. James and Liebeck book, Chapter 17, Exercises 3, 4, 7; Chapter 18, Exercises 2, 3.

2. A representation (ρ, V) of the finite group G is called *self-dual* if it is equivalent to (ρ^*, V^*).

(a) Show that ρ is self-dual if and only if $\chi_\rho(g) \in \mathbb{R}$ for all $g \in G$.

(b) Show that ρ is self-dual if and only if the multiplicity of the trivial representation in $\rho \otimes \rho$ is positive.

(c) Find all the distinct groups of order at most 6 all of whose representations are self-dual.

3. The quaternion group Q_8 has a unique irreducible representation (ρ, V) of dimension 2. The character table was given in problem 4 (c) of the midterm.

(a) Show that every representation of Q_8 is self-dual.

(b) What is the multiplicity of the trivial representation in the representation $\rho \otimes \rho$? What is the multiplicity of ρ in $\rho \otimes \rho$?

(c) Write $\rho \otimes \rho$ and $\rho \otimes \rho \otimes \rho$ as direct sums of irreducible representations of Q_8.