1. James and Liebeck book, Chapter 17, Exercises 3, 4, 7; Chapter 18, Exercises 2, 3.

2. A representation \((\rho, V)\) of the finite group \(G\) is called \textit{self-dual} if it is equivalent to \((\rho^*, V^*)\).
   (a) Show that \(\rho\) is self-dual if and only if \(\chi_\rho(g) \in \mathbb{R}\) for all \(g \in G\).
   (b) Show that \(\rho\) is self-dual if and only if the multiplicity of the trivial representation in \(\rho \otimes \rho\) is positive.
   (c) Find all the distinct groups of order at most 6 all of whose representations are self-dual.

3. The quaternion group \(Q_8\) has a unique irreducible representation \((\rho, V)\) of dimension 2. The character table was given in problem 4 (c) of the midterm.
   (a) Show that every representation of \(Q_8\) is self-dual.
   (b) What is the multiplicity of the trivial representation in the representation \(\rho \otimes \rho\)? What is the multiplicity of \(\rho\) in \(\rho \otimes \rho\)?
   (c) Write \(\rho \otimes \rho\) and \(\rho \otimes \rho \otimes \rho\) as direct sums of irreducible representations of \(Q_8\).