

## REPRESENTATION THEORY W4044

### 1. HOMEWORK, WEEK 1, DUE JANUARY 28

This week's homework is a review of linear algebra. It should be easy.

1. Let  $V$ ,  $V'$ , and  $V''$  be three finite-dimensional vector spaces over a field  $k$ . Let

$$f : V \rightarrow V' \quad g : V' \rightarrow V''$$

be two linear maps. Show that

- (a)  $\dim \operatorname{Im}(f) \cap \operatorname{Ker}(g) = \dim \operatorname{Im}(f) - \dim \operatorname{Im}(g \circ f)$ .
- (b)  $\dim \operatorname{Ker}(g \circ f) \leq \dim \operatorname{Ker}(f) + \dim \operatorname{Ker}(g)$ .

2. Let  $V$  be the vector space over  $\mathbb{R}$  of polynomials with coefficients in  $\mathbb{R}$  of degree at most 3. Consider the map  $f : V \rightarrow V$  defined by

$$f(P)(X) = X^2 P''(X) - (X + 1)P'(X) + 3P(X).$$

In other words,  $f(P)$  is the polynomial in  $X$  defined by the right-hand side of the formula.

- (a) Show that  $f$  is a linear map.
- (b) Show that  $V = \operatorname{Ker}(f) \oplus \operatorname{Im}(f)$ .

3. Exercises 5 and 9, p. 29, James and Liebeck.