1a,b) Suppose we have a minimal polynomial

$x^n + \cdots + a_n$ of $\sqrt{2}+3i$; then the automorphism sending $\sqrt{2} \to -\sqrt{2}$, $i \to -i$ fixes $\mathbb{Q}$. Thus, if $\sqrt{2}+3i$ were roots of the ~~minimal~~ polynomial $+\sqrt{2}+3i$, $-\sqrt{2}-3i$, $\sqrt{2}-3i$. Must also be roots: expanding the polynomial with such roots ~~we~~ see that its coefficients are in $\mathbb{Q}$. Thus, since the ~~above~~ polynomial must divide the minimal polynomial and has coefficients in $\mathbb{Q}$ it is the minimal polynomial.

Thus, the minimal polynomial is:

$$(X-(\sqrt{2}+i3))(X-(-\sqrt{2}-i3))$$

$$(X-(-\sqrt{2}+i3))(X-(\sqrt{2}-i3))$$

(You must expand out and show coefficients are in $\mathbb{Q}$ to get credit.

And since it is of degree 4 The degree of the extension must also be.

2.) Take any non-zero element r. We must show it is invertible.
Consider the set $\{1, r, \ldots, r^n, \ldots\}$.
Since this is a finite dimensional vector space there is some linear dependence relation:

$$c_n r^n + \cdots + c_1 = 0.$$

With $c_i \in K$. Now suppose $c_1 = 0$ then

$$r(c_n r^{n-1} + \cdots + c_2) = 0.$$

Now since R is domain either $r = 0$ or $c_n r^{n-1} + \cdots + c_2 = 0$. By assuming we have choosen the dependence relation of minimal length the latter is impossible. Thus $r = 0$ a contradiction. Hence $c_1 \neq 0$.
Now

$$r(c_n r^{n-1} + \cdots + c_2) = -c_1$$

$$\Rightarrow r(-c_1^{-1}(c_n r^{n-1} + \cdots + c_2)) = 1$$

Thus, r has an inverse.

3a.] Since the degree of field extensions is multiplicative an p is prime either $K' = L$ or $K' = K$ since $[K' : K] = 1$ or p.

b.] We know $[K(\alpha^2) : K] \cdot [K(\alpha) : K(\alpha^2)] = [K(\alpha) : K]$ ~~Now if we~~ wish to consider $[K(\alpha) : K(\alpha^2)]$. Well we know that the minimal polynomial of α with coefficients in $K(\alpha)$ must divide $x^2 - \alpha^2 \in K(\alpha^2)[x]$
Thus, it must be of deg 1 or 2.
Hence, $[K(\alpha^2) : K] = \begin{cases} [K(\alpha) : K] & \text{or} \\ \dfrac{[K(\alpha) : K]}{2} \end{cases}$