Solution to Midterm Practice

1.
(i) Defining a relation on X: R is a relation on X is R is a subset of $X \times X$
(ii) Let X = {1} and R be the empty set. R is symmetric and transitive but not reflexive.
(iii) A possible condition: For any a, b in X, both aRb and bRa are true.

2.

3.
(a) There exists a natural number n such that for any integer a, there exists an integer b such that a = nb.
It is not true.
(b) For any natural number n, there exists an integer a such that for any integer b, a=nb.
It is not true.
(c) There exists a natural number n such that there exists an integer b such that for any integer a, a=nb.
It is not true.
(d) For any integer a, there exists a natural number n such that there exists an integer b, a =nb.
It is true because we can let b=a and n=1 in this case.

4.

5.
(a)
(i)
Z/5Z={[0],[1],[2],[3],[4]}.
f is well defined:

f([0])=[2*0]=[0]
f([1])=[2]
f([2])=[4]
f([3])=[6]=[1]
f([4])=[8]=[3]

All elements in the domain go to a unique element in the range. Therefore, f is well-defined.

(ii) f is injective:
If f([a])=f([b]), then [2a]=[2b], which means that 2a mod 5 = 2b mod 5. Therefore, 3*2a mod 5 = 3*2b mod 5 => (6 mod 5)*(a mod 5) = (6 mod 5)*(b mod 5).
Since 6 mod 5 =1 mod5, we have a mod 5 = b mod 5 => [a] = [b].
Therefore, f([a])=f([b]) => [a]=[b], f is injective.
(iii) f is surjective. Consider any [y] in Z/5Z. Obviously, [3y] is also an element in Z/5Z.
f([3y])=[6y]=[y]. f is surjective.

(b) They are 7 and 7.

6.
(b) It's [0, 1].