INTRODUCTION TO HIGHER MATHEMATICS V2000

Homework, weeks 4-5, due Tuesday October 11 (NOTE THE DATE!)

1. Let $\mathbb{N}$ be the set of positive integers, and let P be the population of the United States. We use the word *smaller* in the usual way; thus if $x, y \in P$ or $n, m \in \mathbb{N}$ one can make the statements x *is smaller than* y or n *is smaller than* m. In particular, we want to define relations R on $\mathbb{N}$ or on P by setting xRy to mean x *is not smaller than* y if $x, y \in P$ and nRm to mean n *is not smaller than* m if $n, m \in \mathbb{N}$.

(a) Explain informally why in both cases R should be a linear ordering. We assume in what follows that R has this property.

(b) In what follows X is either $\mathbb{N}$ or P. Consider the statements

a. $\forall x \in X \exists y \in X \ yRx$.
b. $\exists y \in X \forall x \in X \ yRx$.

(i) Translate sentences (a) and (b) into English and interpret each of them as a sentence about $\mathbb{N}$ and as a sentence about P.

(ii) One of these four sentences is impossible. Which sentence is it, and why is it impossible?

(c) (From Dumas-McCarthy, Exercise 3.16). Let X be any set, and let $P(x, y)$ be a formula in two variables. What is the relation between the sentence $\exists y \in X \exists x \in X \ P(x, y)$ and $\exists x \in X \exists y \in X \ P(x, y)$?

2. Dumas McCarthy, Exercises 3.9, 3.10, 3.13, 3.14 (parts (c) and (d) only), 3.17, and the second part of 3.5: prove that there are statements using only P, Q, $\neq$, and $\vee$ that are equivalent to (a), (b), (c).

3. Some review problems from Daepp and Gorkin: do as many of the following exercises as you can, and hand in any *three* of them.
Problems 8.8-8.9, 8.16, 8.18, 10.7, 10.8, 17.13.