

INTRODUCTION TO HIGHER MATHEMATICS V2000

HOMEWORK, WEEK 3, DUE SEPTEMBER 29

From Dumas, McCarthy, Second Edition, Chapters 2 and 3.

1. In this exercise angles are expressed in radians. Consider the relation on angles $\alpha \sim \alpha'$ if $\alpha - \alpha'$ is an integer multiple of 2π .

(a) Show that this is an equivalence relation (cf. Example 2.23 from Dumas-McCarthy).

(b) Let α and β be two angles. Using the familiar trigonometric formulas for $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$, show that,

$$\alpha \sim \alpha' \Rightarrow \sin(\alpha + \beta) = \sin(\alpha' + \beta) \forall \beta;$$

$$\beta \sim \beta' \Rightarrow \cos(\alpha + \beta) = \cos(\alpha + \beta') \forall \alpha.$$

2. Exercises 2.15, 2.16, 2.19. (Review Examples 2.33 and 2.34.)

3. (a) Exercise 2.17 in the following form: Let n be a positive integer and let $[n]$ be its residue modulo 3. Let m denote the sum of the digits of n ; e.g., if $n = 451$ then $m = 4 + 5 + 1$; and let $[m]$ be the residue of m modulo 3. Show that $[m] = [n]$.

(b) (Optional) Do Exercise 2.18.

4. Exercises 2.22, 2.26, 3.1 part (i), 3.2, 3.3, 3.4