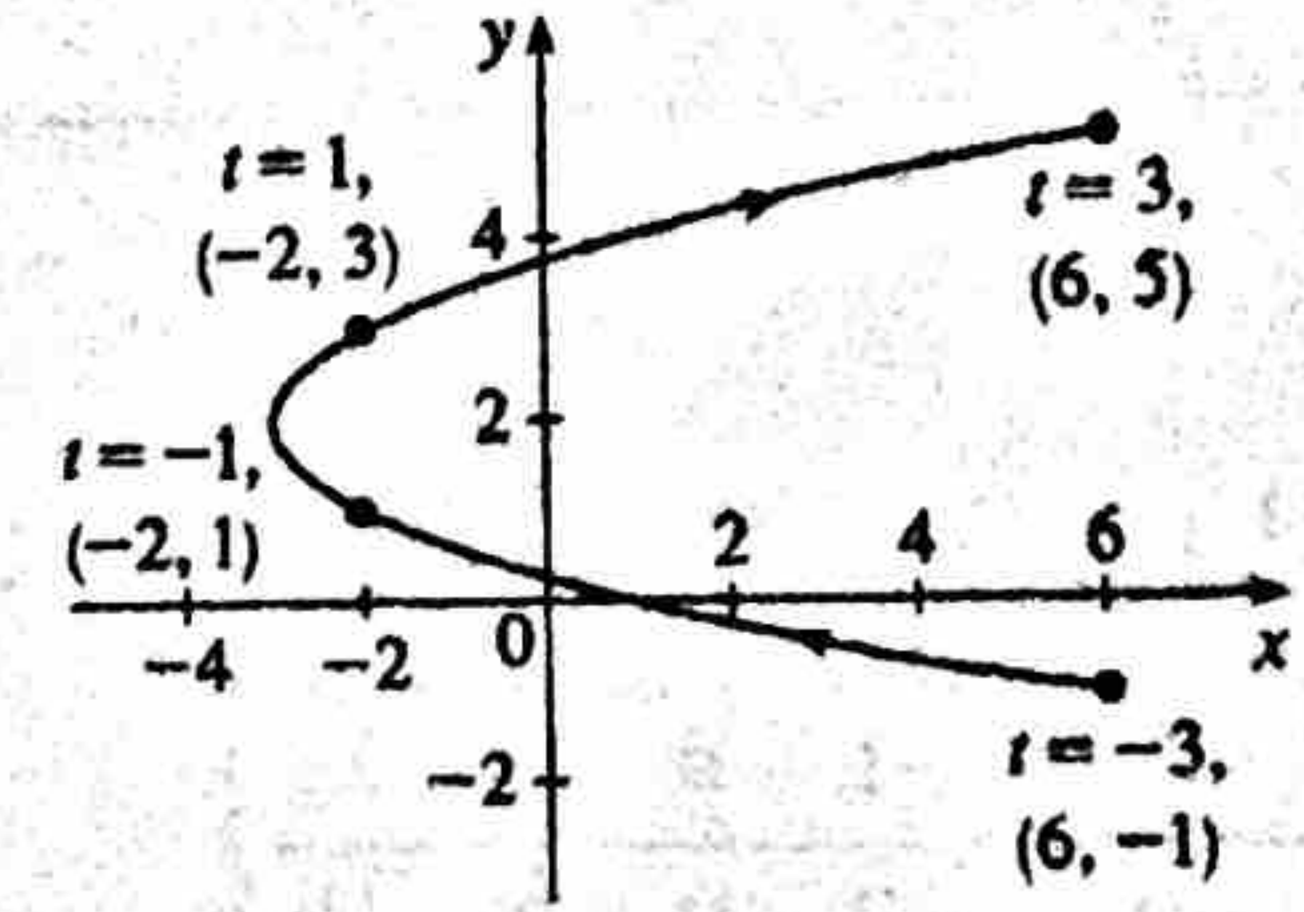
7. $x = t^2 - 3, \quad y = t + 2, \quad -3 \le t \le 3$

(a)

t	-3	-1	1	3
x	6	-2	-2	6
y	-1	1	3	5



(b) $y = t + 2 \quad \Rightarrow \quad t = y - 2$, so

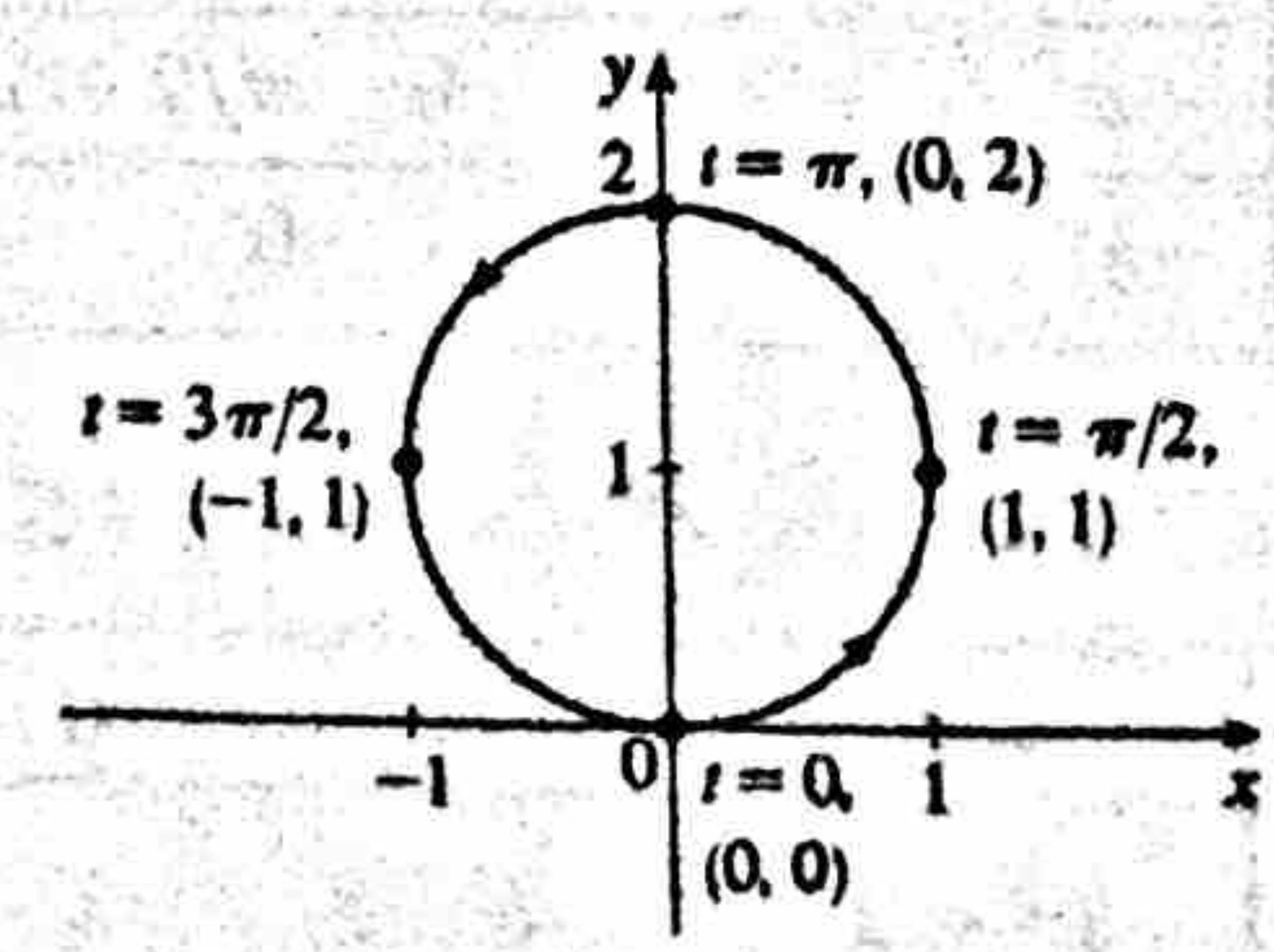
$x = t^2 - 3 = (y-2)^2 - 3 = y^2 - 4y + 4 - 3 \quad \Rightarrow$

$x = y^2 - 4y + 1, -1 \le y \le 5$

8. $x = \sin t, \quad y = 1 - \cos t, \quad 0 \le t \le 2\pi$

(a)

t	0	$\pi/2$	π	$3\pi/2$	2π
x	0	1	0	-1	0
y	0	1	2	1	0



(b) $x = \sin t, y = 1 - \cos t \quad [\text{or } y - 1 = -\cos t] \quad \Rightarrow$

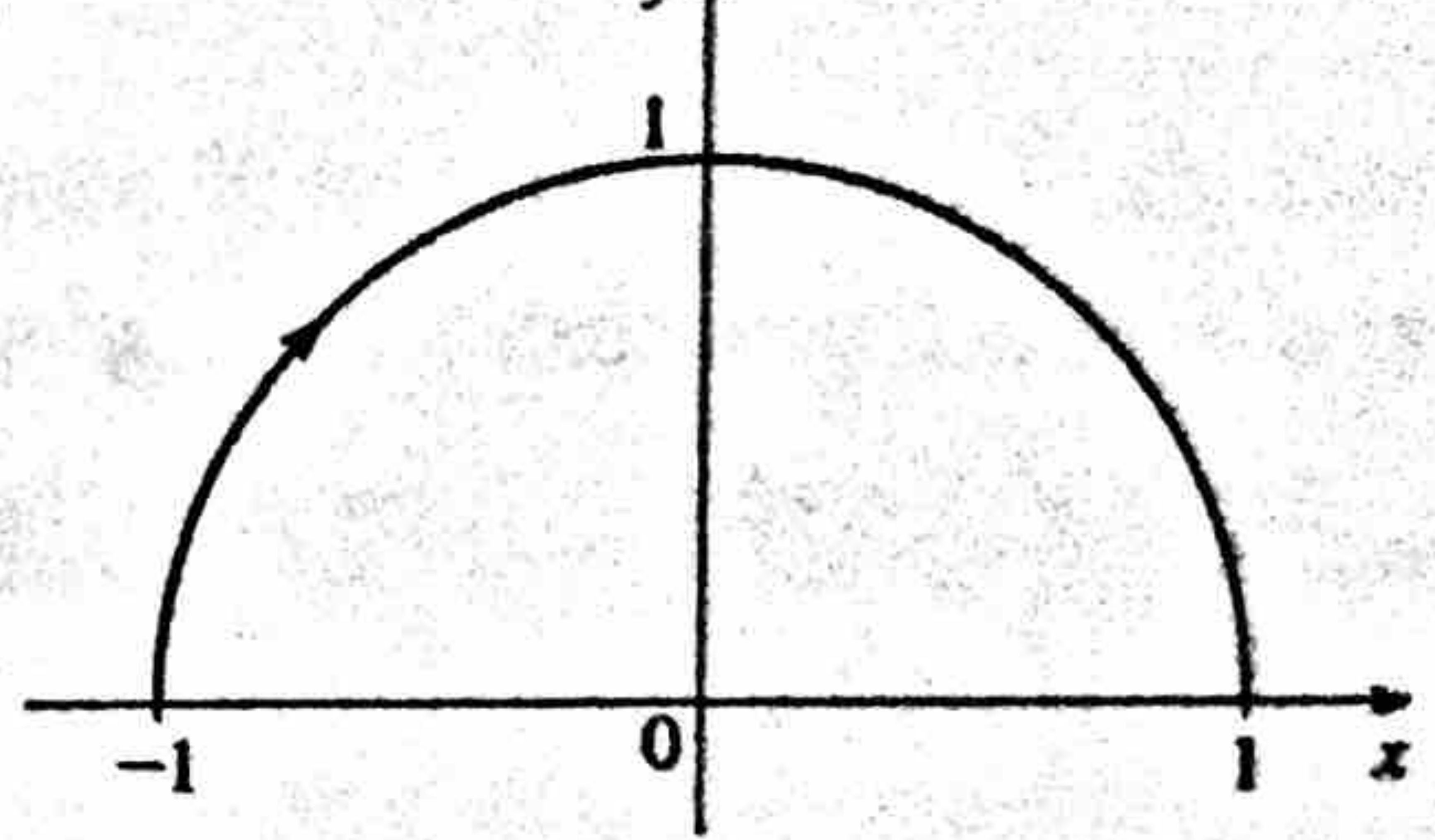
$x^2 + (y-1)^2 = (\sin t)^2 + (-\cos t)^2 \quad \Rightarrow \quad x^2 + (y-1)^2 = 1.$

As t varies from 0 to 2π, the circle with center $(0, 1)$ and radius 1 is traced out.

11. (a) $x = \sin \frac{1}{2}\theta$, $y = \cos \frac{1}{2}\theta$, $-\pi \le \theta \le \pi$.

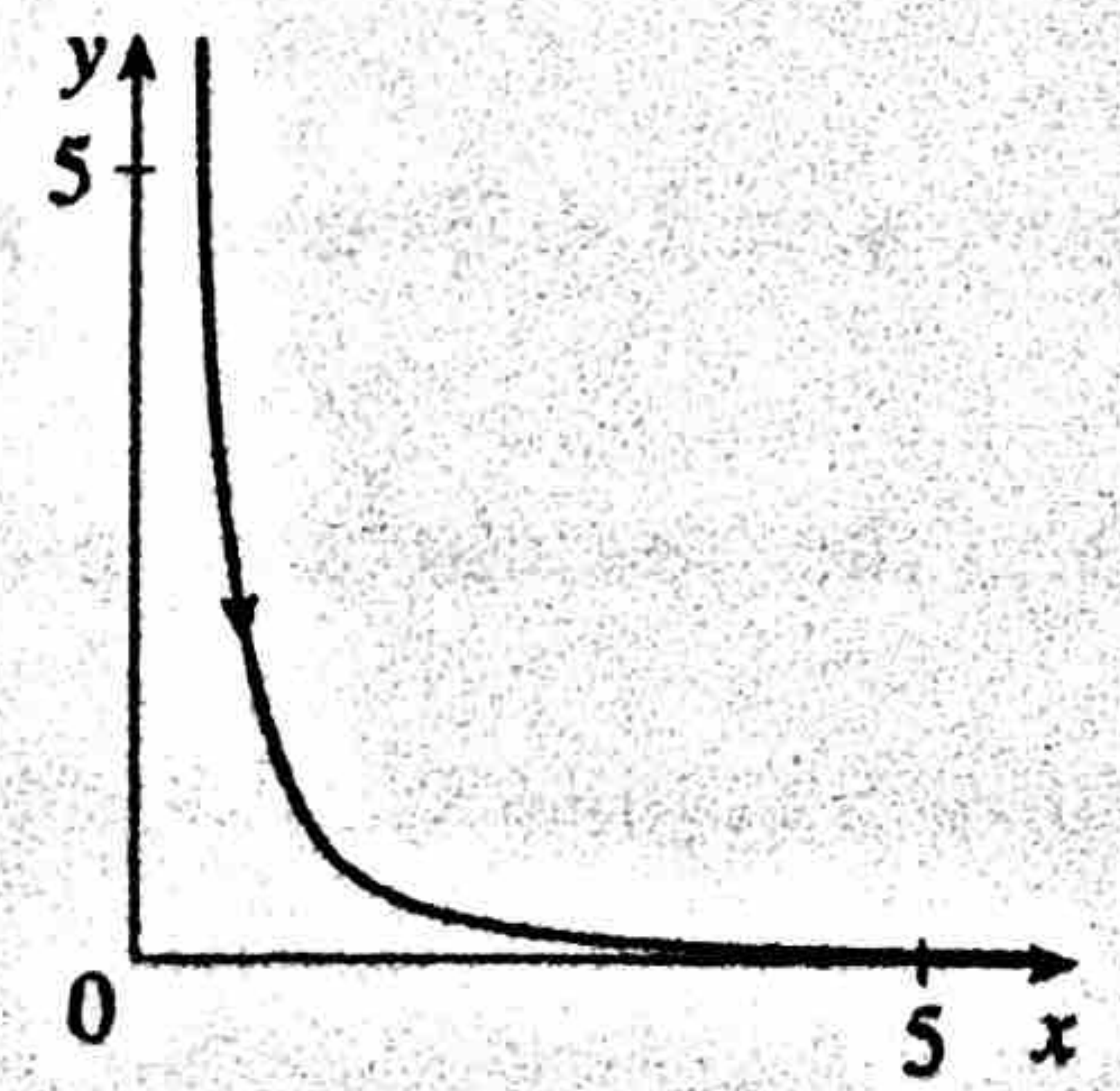
$x^2 + y^2 = \sin^2 \frac{1}{2}\theta + \cos^2 \frac{1}{2}\theta = 1$. For $-\pi \le \theta \le 0$, we have $-1 \le x \le 0$ and $0 \le y \le 1$. For $0 < \theta \le \pi$, we have $0 < x \le 1$ and $1 > y \ge 0$. The graph is a semicircle.

(b)



14. (a) $y = e^{-2t} = (e^t)^{-2} = x^{-2} = 1/x^2$ for $x > 0$ since $x = e^t$.

(b)



24. (a) From the first graph, we have $1 \leq x \leq 2$. From the second graph, we have $-1 \leq y \leq 1$. The only choice that satisfies either of those conditions is III.

(b) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

(c) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph, we have $0 \leq y \leq 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y do the same thing. Choice II satisfies these conditions.

28\. (a) $x = t^4 - t + 1 = (t^4 + 1) - t > 0$ [think of the graphs of $y = t^4 + 1$ and $y = t$] and $y = t^2 \geq 0$, so these equations are matched with graph V.

(b) $y = \sqrt{t} \geq 0$. $x = t^2 - 2t = t(t-2)$ is negative for $0 < t < 2$, so these equations are matched with graph I.

(c) $x = \sin 2t$ has period $2\pi/2 = \pi$. Note that

$y(t + 2\pi) = \sin[t + 2\pi + \sin 2(t + 2\pi)] = \sin(t + 2\pi + \sin 2t) = \sin(t + \sin 2t) = y(t)$, so y has period 2π. These equations match graph II since x cycles through the values -1 to 1 twice as y cycles through those values once.

(d) $x = \cos 5t$ has period $2\pi/5$ and $y = \sin 2t$ has period π, so x will take on the values -1 to 1, and then 1 to -1, before y takes on the values -1 to 1. Note that when $t = 0$, $(x, y) = (1, 0)$. These equations are matched with graph VI.

(e) $x = t + \sin 4t$, $y = t^2 + \cos 3t$. As t becomes large, t and t^2 become the dominant terms in the expressions for x and y, so the graph will look like the graph of $y = x^2$, but with oscillations. These equations are matched with graph IV.

(f) $x = \dfrac{\sin 2t}{4 + t^2}$, $y = \dfrac{\cos 2t}{4 + t^2}$. As $t \to \infty$, x and y both approach 0. These equations are matched with graph III.

1. The component functions $\ln(t+1)$, $\dfrac{t}{\sqrt{9-t^2}}$, and 2^t are all defined when $t+1>0 \Rightarrow t>-1$ and $9-t^2>0 \Rightarrow$ $-3<t<3$, so the domain of $\mathbf{r}$ is $(-1,3)$.

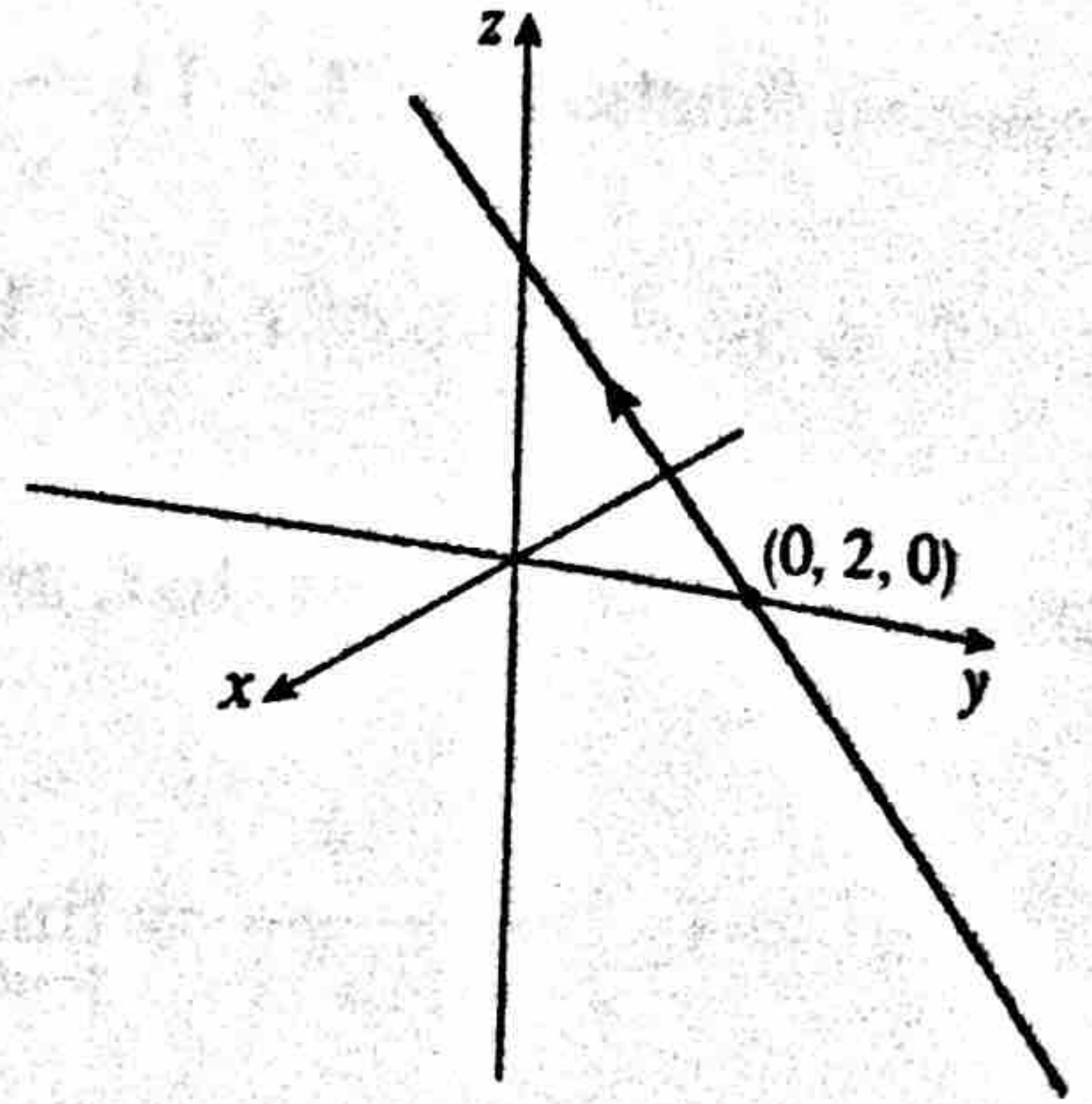
4. $\lim_{t\to 1} \frac{t^2-t}{t-1} = \lim_{t\to 1} \frac{t(t-1)}{t-1} = \lim_{t\to 1} t = 1$, $\lim_{t\to 1} \sqrt{t+8} = 3$, $\lim_{t\to 1} \frac{\sin \pi t}{\ln t} = \lim_{t\to 1} \frac{\pi \cos \pi t}{1/t} = -\pi$ [by l'Hospital's Rule].

Thus the given limit equals $\mathbf{i} + 3\,\mathbf{j} - \pi\,\mathbf{k}$.

5. $\lim_{t\to\infty} \frac{1+t^2}{1-t^2} = \lim_{t\to\infty} \frac{(1/t^2)+1}{(1/t^2)-1} = \frac{0+1}{0-1} = -1$, $\lim_{t\to\infty} \tan^{-1} t = \frac{\pi}{2}$, $\lim_{t\to\infty} \frac{1-e^{-2t}}{t} = \lim_{t\to\infty} \frac{1}{t} - \frac{1}{te^{2t}} = 0 - 0 = 0$. Thus

$$\lim_{t\to\infty} \left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle = \left\langle -1, \frac{\pi}{2}, 0 \right\rangle.$$

9. The corresponding parametric equations are $x = t,\ y = 2 - t,\ z = 2t$, which are parametric equations of a line through the point $(0, 2, 0)$ and with direction vector $\langle 1, -1, 2 \rangle$.



10. The corresponding parametric equations are $x = \sin \pi t,\ y = t,\ z = \cos \pi t$. Note that $x^2 + z^2 = \sin^2 \pi t + \cos^2 \pi t = 1$, so the curve lies on the circular cylinder $x^2 + z^2 = 1$. A point (x, y, z) on the curve lies directly to the left or right of the point $(x, 0, z)$ which moves clockwise (when viewed from the left) along the circle $x^2 + z^2 = 1$ in the xz-plane as t increases. Since $y = t$, the curve is a helix that spirals toward the right around the cylinder.