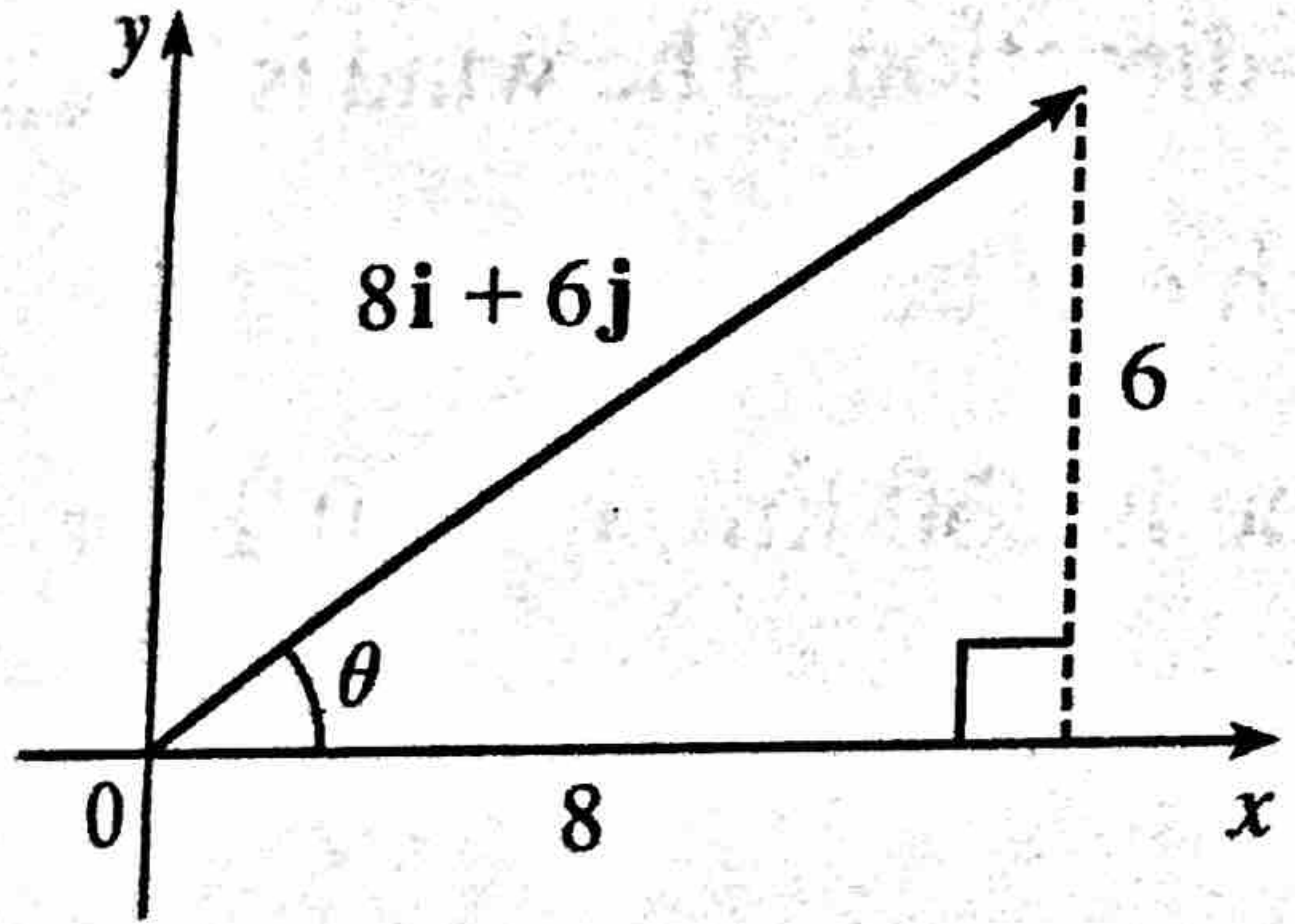
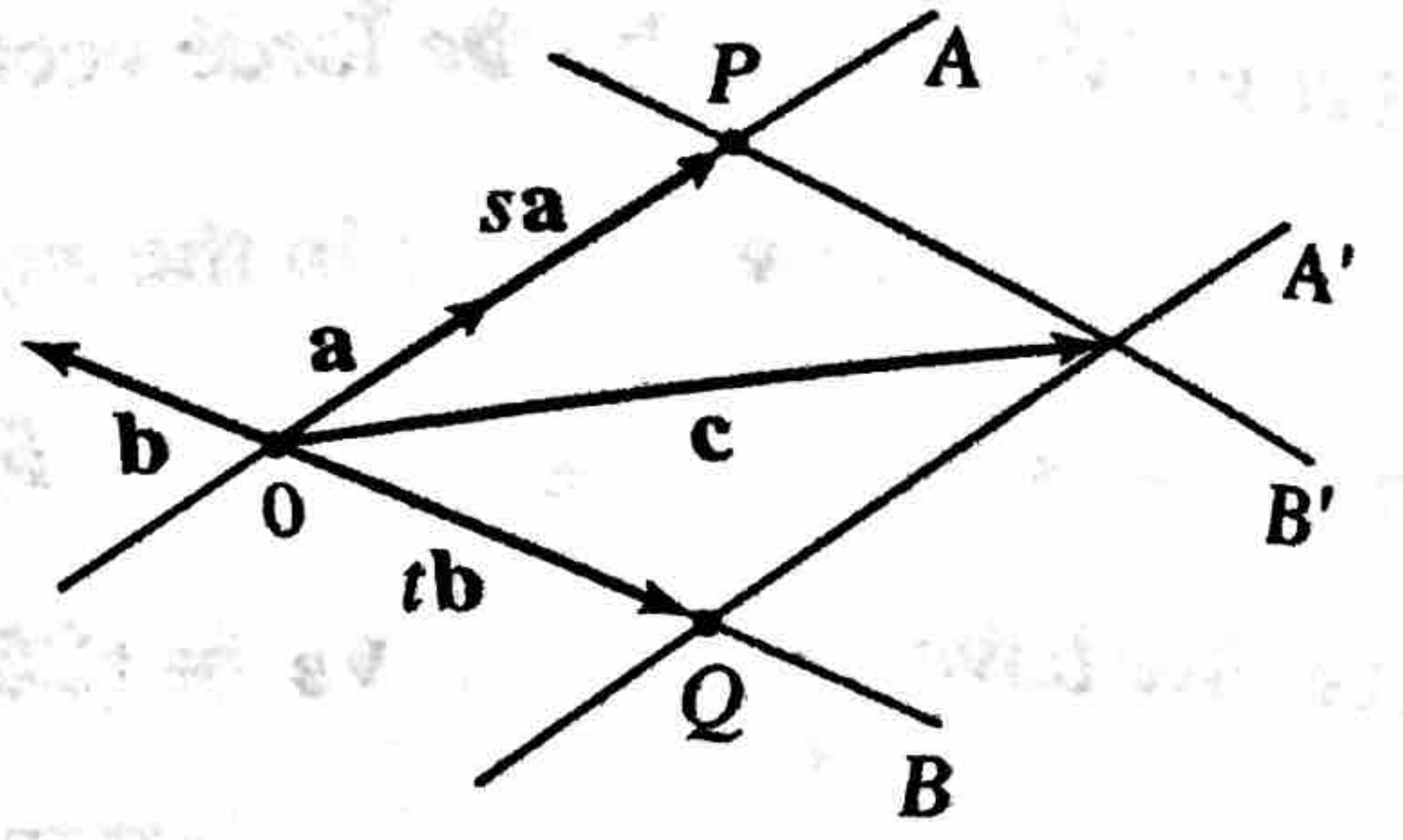
24. The vector $-5\,\mathbf{i} + 3\,\mathbf{j} - \mathbf{k}$ has length $|-5\,\mathbf{i} + 3\,\mathbf{j} - \mathbf{k}| = \sqrt{(-5)^2 + 3^2 + (-1)^2} = \sqrt{35}$, so by Equation 4 the unit vector with the same direction is $\dfrac{1}{\sqrt{35}}(-5\,\mathbf{i} + 3\,\mathbf{j} - \mathbf{k}) = -\dfrac{5}{\sqrt{35}}\,\mathbf{i} + \dfrac{3}{\sqrt{35}}\,\mathbf{j} - \dfrac{1}{\sqrt{35}}\,\mathbf{k}$.

28. 

From the figure we see that $\tan\theta = \frac{6}{8} = \frac{3}{4}$, so $\theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9^\circ$.

44. $\overrightarrow{AC} = \frac{1}{3}\overrightarrow{AB}$ and $\overrightarrow{BC} = \frac{2}{3}\overrightarrow{BA}$. $\mathbf{c} = \overrightarrow{OA} + \overrightarrow{AC} = \mathbf{a} + \frac{1}{3}\overrightarrow{AB} \;\Rightarrow\; \overrightarrow{AB} = 3\,\mathbf{c} - 3\,\mathbf{a}$. $\mathbf{c} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{BA} \;\Rightarrow$ $\overrightarrow{BA} = \frac{3}{2}\,\mathbf{c} - \frac{3}{2}\,\mathbf{b}$. $\overrightarrow{BA} = -\overrightarrow{AB}$, so $\frac{3}{2}\,\mathbf{c} - \frac{3}{2}\,\mathbf{b} = 3\,\mathbf{a} - 3\,\mathbf{c} \;\Leftrightarrow\; \mathbf{c} + 2\,\mathbf{c} = 2\,\mathbf{a} + \mathbf{b} \;\Leftrightarrow\; \mathbf{c} = \frac{2}{3}\,\mathbf{a} + \frac{1}{3}\,\mathbf{b}$.

46. Draw **a**, **b**, and **c** emanating from the origin. Extend **a** and **b** to form lines A and B, and draw lines A' and B' parallel to these two lines through the terminal point of **c**. Since **a** and **b** are not parallel, A and B' must meet (at P), and A' and B must also meet (at Q). Now we see that $\overrightarrow{OP} + \overrightarrow{OQ} = \mathbf{c}$, so if



$s = \dfrac{|\overrightarrow{OP}|}{|\mathbf{a}|}$ (or its negative, if **a** points in the direction opposite $\overrightarrow{OP}$) and $t = \dfrac{|\overrightarrow{OQ}|}{|\mathbf{b}|}$ (or its negative, as in the diagram), then $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$, as required.

Argument using components: Since **a**, **b**, and **c** all lie in the same plane, we can consider them to be vectors in two dimensions. Let $\mathbf{a} = \langle a_1, a_2 \rangle$, $\mathbf{b} = \langle b_1, b_2 \rangle$, and $\mathbf{c} = \langle c_1, c_2 \rangle$. We need $sa_1 + tb_1 = c_1$ and $sa_2 + tb_2 = c_2$. Multiplying the first equation by a_2 and the second by a_1 and subtracting, we get $t = \dfrac{c_2 a_1 - c_1 a_2}{b_2 a_1 - b_1 a_2}$. Similarly $s = \dfrac{b_2 c_1 - b_1 c_2}{b_2 a_1 - b_1 a_2}$. Since $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{0}$ and **a** is not a scalar multiple of **b**, the denominator is not zero.

1. (a) $\mathbf{a}\cdot\mathbf{b}$ is a scalar, and the dot product is defined only for vectors, so $(\mathbf{a}\cdot\mathbf{b})\cdot\mathbf{c}$ has no meaning.

(b) $(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}$ is a scalar multiple of a vector, so it does have meaning.

(c) Both $|\mathbf{a}|$ and $\mathbf{b}\cdot\mathbf{c}$ are scalars, so $|\mathbf{a}|\,(\mathbf{b}\cdot\mathbf{c})$ is an ordinary product of real numbers, and has meaning.

(d) Both $\mathbf{a}$ and $\mathbf{b}+\mathbf{c}$ are vectors, so the dot product $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})$ has meaning.

(e) $\mathbf{a}\cdot\mathbf{b}$ is a scalar, but $\mathbf{c}$ is a vector, and so the two quantities cannot be added and $\mathbf{a}\cdot\mathbf{b}+\mathbf{c}$ has no meaning.

(f) $|\mathbf{a}|$ is a scalar, and the dot product is defined only for vectors, so $|\mathbf{a}|\cdot(\mathbf{b}+\mathbf{c})$ has no meaning.

2. $\mathbf{a} \cdot \mathbf{b} = \langle 5, -2 \rangle \cdot \langle 3, 4 \rangle = (5)(3) + (-2)(4) = 15 - 8 = 7$

5. $\mathbf{a} \cdot \mathbf{b} = \left\langle 4, 1, \frac{1}{4} \right\rangle \cdot \langle 6, -3, -8 \rangle = (4)(6) + (1)(-3) + \left(\frac{1}{4}\right)(-8) = 19$

$(2\mathbf{i} + \mathbf{j}) \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k}) = (2)(1) + (1)(-1) + (0)(1)$

8. $\mathbf{a} \cdot \mathbf{b} = (3\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot (4\mathbf{i} + 5\mathbf{k}) = (3)(4) + (2)(0) + (-1)(5) = 7$

9. By Theorem 3, $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta = (7)(4) \cos 30° = 28\left(\frac{\sqrt{3}}{2}\right) = 14$

17. $|\mathbf{a}| = \sqrt{1^2 + (-4)^2 + 1^2} = \sqrt{18} = 3\sqrt{2}$, $|\mathbf{b}| = \sqrt{0^2 + 2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$, and

$\mathbf{a} \cdot \mathbf{b} = (1)(0) + (-4)(2) + (1)(-2) = -10$. From Corollary 6, we have $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{-10}{3\sqrt{2} \cdot 2\sqrt{2}} = -\dfrac{10}{12} =$

the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\theta = \cos^{-1}\left(-\frac{5}{6}\right) \approx 146°$.

23. (a) $\mathbf{a} \cdot \mathbf{b} = (9)(-2) + (3)(6) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

(b) $\mathbf{a} \cdot \mathbf{b} = (4)(3) + (5)(-1) + (-2)(5) = -3 \neq 0$, so $\mathbf{a}$ and $\mathbf{b}$ are not orthogonal. Also, since $\mathbf{a}$ is not a scalar multiple of $\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

(c) $\mathbf{a} \cdot \mathbf{b} = (-8)(6) + (12)(-9) + (4)(-3) = -168 \neq 0$, so $\mathbf{a}$ and $\mathbf{b}$ are not orthogonal. Because $\mathbf{a} = -\frac{4}{3}\,\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are parallel.

(d) $\mathbf{a} \cdot \mathbf{b} = (3)(5) + (-1)(9) + (3)(-2) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

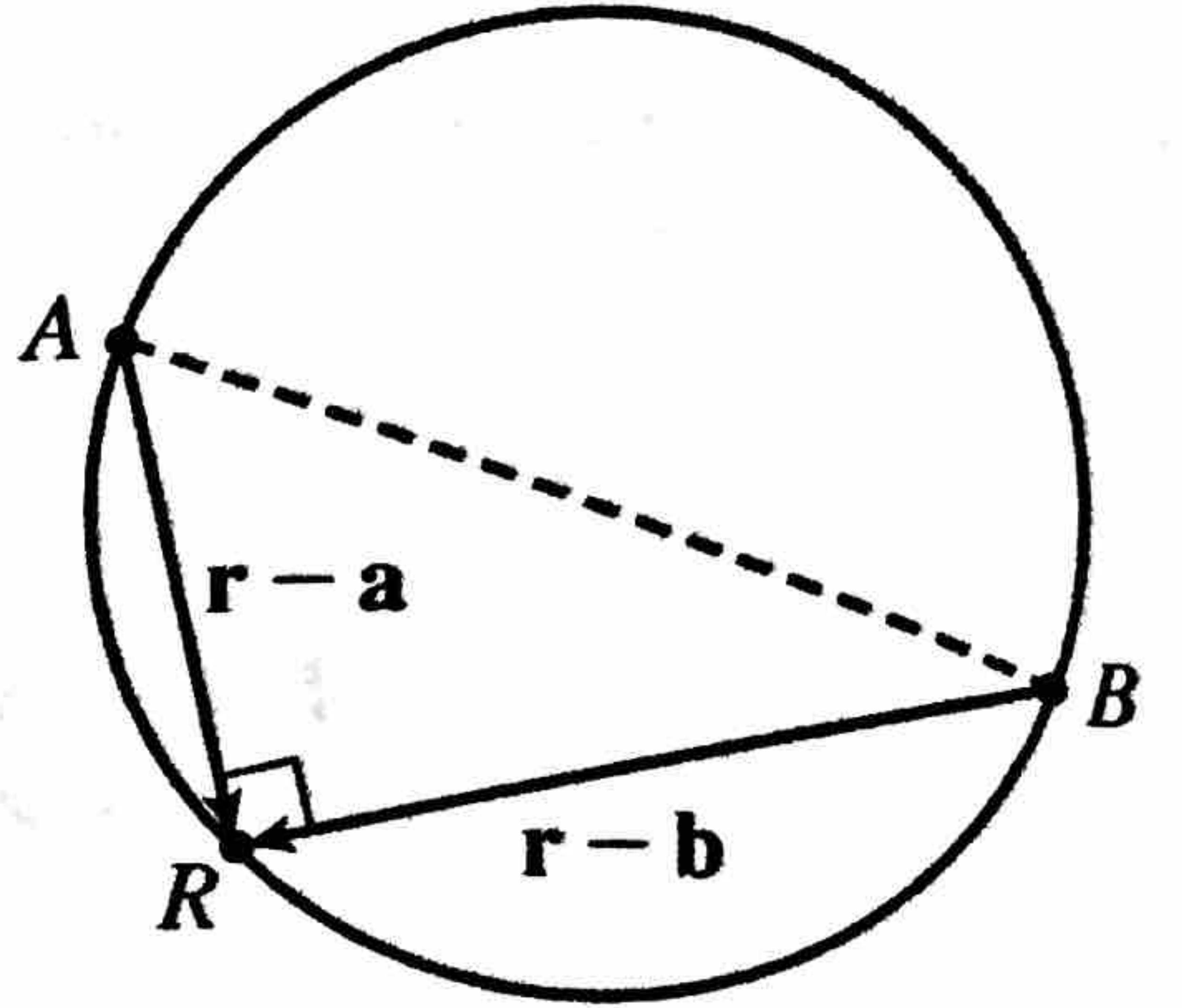
31. The curves $y = x^2$ and $y = x^3$ meet when $x^2 = x^3 \Leftrightarrow x^3 - x^2 = 0 \Leftrightarrow x^2(x-1) = 0 \Leftrightarrow x = 0, x = 1$. We have $\frac{d}{dx}x^2 = 2x$ and $\frac{d}{dx}x^3 = 3x^2$, so the tangent lines of both curves have slope 0 at $x = 0$. Thus the angle between the curves is $0°$ at the point $(0,0)$. For $x = 1$, $\left.\frac{d}{dx}\,x^2\right|_{x=1} = 2$ and $\left.\frac{d}{dx}\,x^3\right|_{x=1} = 3$ so the tangent lines at the point $(1,1)$ have slopes 2 and 3. Vectors parallel to the tangent lines are $\langle 1,2\rangle$ and $\langle 1,3\rangle$, and the angle θ between them is given by

$$\cos\theta = \frac{\langle 1,2\rangle \cdot \langle 1,3\rangle}{|\langle 1,2\rangle|\,|\langle 1,3\rangle|} = \frac{1+6}{\sqrt{5}\sqrt{10}} = \frac{7}{5\sqrt{2}}$$

Thus $\theta = \cos^{-1}\left(\frac{7}{5\sqrt{2}}\right) \approx 8.1°$.

43. $|\mathbf{a}| = \sqrt{9+9+1} = \sqrt{19}$ so the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{6-12-1}{\sqrt{19}} = -\dfrac{7}{\sqrt{19}}$ while the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{proj}_{\mathbf{a}}\,\mathbf{b} = -\dfrac{7}{\sqrt{19}}\dfrac{\mathbf{a}}{|\mathbf{a}|} = -\frac{7}{\sqrt{19}}\cdot\frac{1}{\sqrt{19}}(3\,\mathbf{i}-3\,\mathbf{j}+\mathbf{k}) = -\frac{7}{19}(3\,\mathbf{i}-3\,\mathbf{j}+\mathbf{k}) = -\frac{21}{19}\,\mathbf{i}+\frac{21}{19}\,\mathbf{j}-\frac{7}{19}\,\mathbf{k}$.

54. $(\mathbf{r}-\mathbf{a})\cdot(\mathbf{r}-\mathbf{b})=0$ implies that the vectors $\mathbf{r}-\mathbf{a}$ and $\mathbf{r}-\mathbf{b}$ are orthogonal. From the diagram (in which A, B and R are the terminal points of the vectors), we see that this implies that R lies on a sphere whose diameter is the line from A to B. The center of this circle is the midpoint of AB, that is, $\frac{1}{2}(\mathbf{a}+\mathbf{b})=\left\langle \frac{1}{2}(a_1+b_1), \frac{1}{2}(a_2+b_2), \frac{1}{2}(a_3+b_3)\right\rangle$, and its radius is $\frac{1}{2}|\mathbf{a}-\mathbf{b}|=\frac{1}{2}\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$.



Or: Expand the given equation, substitute $\mathbf{r}\cdot\mathbf{r}=x^2+y^2+z^2$ and complete the squares.

64. If the vectors $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are orthogonal then $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})=0$. But

$$\begin{aligned}(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) &= (\mathbf{u}+\mathbf{v})\cdot\mathbf{u}-(\mathbf{u}+\mathbf{v})\cdot\mathbf{v} && \text{by Property 3 of the dot product}\\ &= \mathbf{u}\cdot\mathbf{u}+\mathbf{v}\cdot\mathbf{u}-\mathbf{u}\cdot\mathbf{v}-\mathbf{v}\cdot\mathbf{v} && \text{by Property 3}\\ &= |\mathbf{u}|^2+\mathbf{u}\cdot\mathbf{v}-\mathbf{u}\cdot\mathbf{v}-|\mathbf{v}|^2 && \text{by Properties 1 and 2}\\ &= |\mathbf{u}|^2-|\mathbf{v}|^2\end{aligned}$$

Thus $|\mathbf{u}|^2-|\mathbf{v}|^2=0 \quad\Rightarrow\quad |\mathbf{u}|^2=|\mathbf{v}|^2 \quad\Rightarrow\quad |\mathbf{u}|=|\mathbf{v}|$ [since $|\mathbf{u}|, |\mathbf{v}|\geq 0$].

4. $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -3 \\ 3 & -3 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -3 \\ -3 & 3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 3 & -3 \\ 3 & 3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 3 & 3 \\ 3 & -3 \end{vmatrix} \mathbf{k}$

$$= (9 - 9)\,\mathbf{i} - [9 - (-9)]\,\mathbf{j} + (-9 - 9)\,\mathbf{k} = -18\,\mathbf{j} - 18\,\mathbf{k}$$

Since $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = (-18\,\mathbf{j} - 18\,\mathbf{k}) \cdot (3\,\mathbf{i} + 3\,\mathbf{j} - 3\,\mathbf{k}) = 0 - 54 + 54 = 0$, $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$.

Since $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = (-18\,\mathbf{j} - 18\,\mathbf{k}) \cdot (3\,\mathbf{i} - 3\,\mathbf{j} + 3\,\mathbf{k}) = 0 + 54 - 54 = 0$, $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{b}$.

6. $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ t & \cos t & \sin t \\ 1 & -\sin t & \cos t \end{vmatrix} = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} \mathbf{i} - \begin{vmatrix} t & \sin t \\ 1 & \cos t \end{vmatrix} \mathbf{j} + \begin{vmatrix} t & \cos t \\ 1 & -\sin t \end{vmatrix} \mathbf{k}$

$$= [\cos^2 t - (-\sin^2 t)]\,\mathbf{i} - (t\cos t - \sin t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k} = \mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k}$$

Since

$$\begin{aligned}(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} &= [\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k}] \cdot (t\,\mathbf{i} + \cos t\,\mathbf{j} + \sin t\,\mathbf{k}) \\ &= t + \sin t\cos t - t\cos^2 t - t\sin^2 t - \sin t\cos t \\ &= t - t\left(\cos^2 t + \sin^2 t\right) = 0\end{aligned}$$

$\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$.

Since

$$\begin{aligned}(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} &= [\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k}] \cdot (\mathbf{i} - \sin t\,\mathbf{j} + \cos t\,\mathbf{k}) \\ &= 1 - \sin^2 t + t\sin t\cos t - t\sin t\cos t - \cos^2 t \\ &= 1 - \left(\sin^2 t + \cos^2 t\right) = 0\end{aligned}$$

$\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{b}$.

13. (a) Since $\mathbf{b} \times \mathbf{c}$ is a vector, the dot product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is meaningful and is a scalar.

(b) $\mathbf{b} \cdot \mathbf{c}$ is a scalar, so $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$ is meaningless, as the cross product is defined only for two *vectors*.

(c) Since $\mathbf{b} \times \mathbf{c}$ is a vector, the cross product $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ is meaningful and results in another vector.

(d) $\mathbf{b} \cdot \mathbf{c}$ is a scalar, so the dot product $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$ is meaningless, as the dot product is defined only for two vectors.

(e) Since $(\mathbf{a} \cdot \mathbf{b})$ and $(\mathbf{c} \cdot \mathbf{d})$ are both scalars, the cross product $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$ is meaningless.

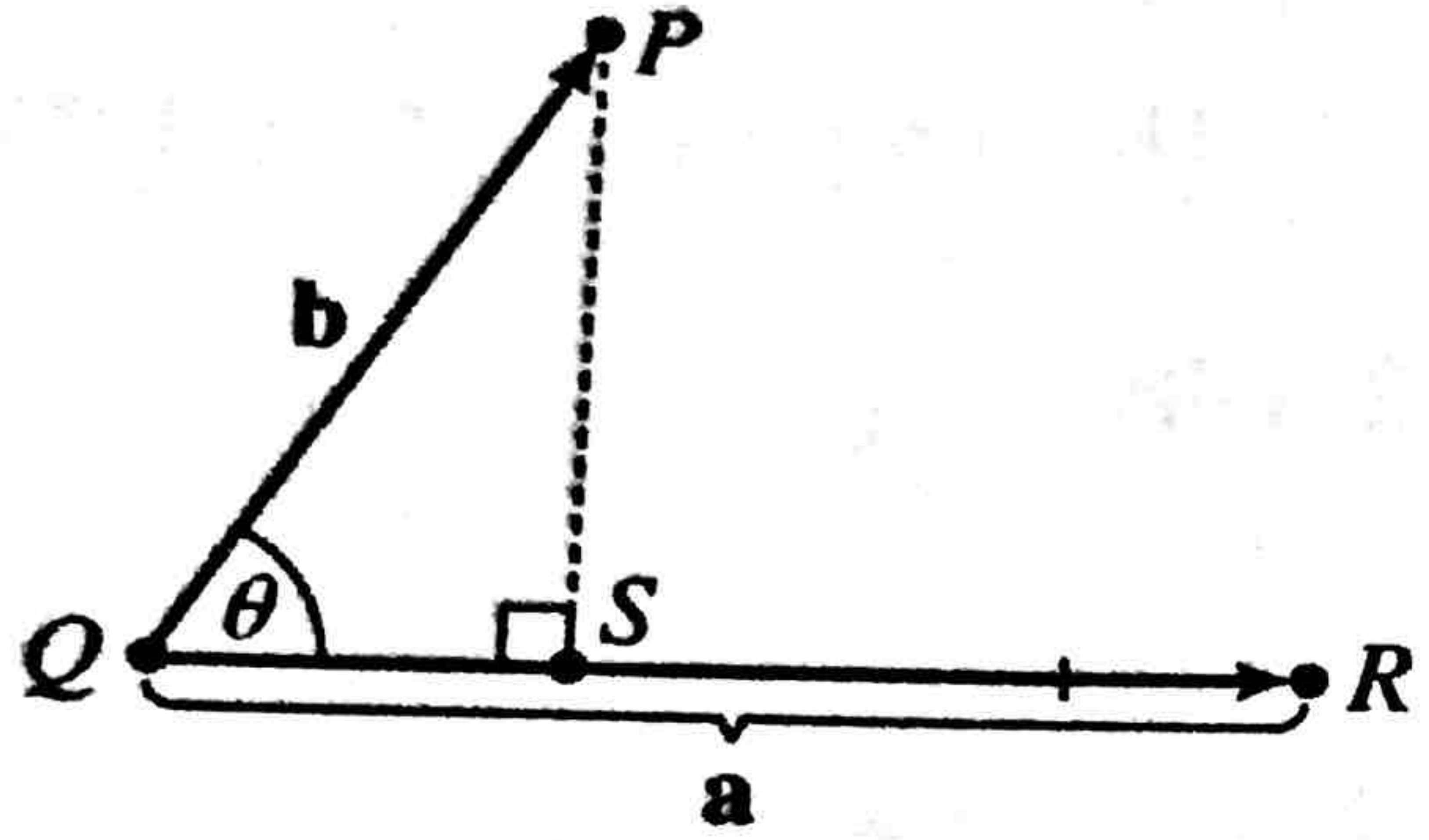
(f) $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c} \times \mathbf{d}$ are both vectors, so the dot product $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$ is meaningful and is a scalar.

14. Using Theorem 9, we have $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta = (4)(5)\sin 45^\circ = 20 \cdot \frac{\sqrt{2}}{2} = 10\sqrt{2}$. By the right-hand rule, $\mathbf{u} \times \mathbf{v}$ is directed out of the page.

25. $$\begin{aligned}\mathbf{a}\times(\mathbf{b}+\mathbf{c}) &= \mathbf{a}\times\langle b_1+c_1, b_2+c_2, b_3+c_3\rangle \\ &= \langle a_2(b_3+c_3)-a_3(b_2+c_2), a_3(b_1+c_1)-a_1(b_3+c_3), a_1(b_2+c_2)-a_2(b_1+c_1)\rangle \\ &= \langle a_2b_3+a_2c_3-a_3b_2-a_3c_2, a_3b_1+a_3c_1-a_1b_3-a_1c_3, a_1b_2+a_1c_2-a_2b_1-a_2c_1\rangle \\ &= \langle (a_2b_3-a_3b_2)+(a_2c_3-a_3c_2), (a_3b_1-a_1b_3)+(a_3c_1-a_1c_3), (a_1b_2-a_2b_1)+(a_1c_2-a_2c_1)\rangle \\ &= \langle a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1\rangle + \langle a_2c_3-a_3c_2, a_3c_1-a_1c_3, a_1c_2-a_2c_1\rangle \\ &= (\mathbf{a}\times\mathbf{b})+(\mathbf{a}\times\mathbf{c})\end{aligned}$$

solution).

45. (a)



The distance between a point and a line is the length of the perpendicular from the point to the line, here $\left|\overrightarrow{PS}\right| = d$. But referring to triangle PQS, $d = \left|\overrightarrow{PS}\right| = \left|\overrightarrow{QP}\right| \sin\theta = |\mathbf{b}| \sin\theta$. But θ is the angle between $\overrightarrow{QP} = \mathbf{b}$ and $\overrightarrow{QR} = \mathbf{a}$. Thus by Theorem 9, $\sin\theta = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|\,|\mathbf{b}|}$ and so $d = |\mathbf{b}| \sin\theta = \dfrac{|\mathbf{b}|\,|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$.

(b) $\mathbf{a} = \overrightarrow{QR} = \langle -1, -2, -1 \rangle$ and $\mathbf{b} = \overrightarrow{QP} = \langle 1, -5, -7 \rangle$. Then

$$\mathbf{a} \times \mathbf{b} = \langle (-2)(-7) - (-1)(-5), (-1)(1) - (-1)(-7), (-1)(-5) - (-2)(1) \rangle = \langle 9, -8, 7 \rangle.$$

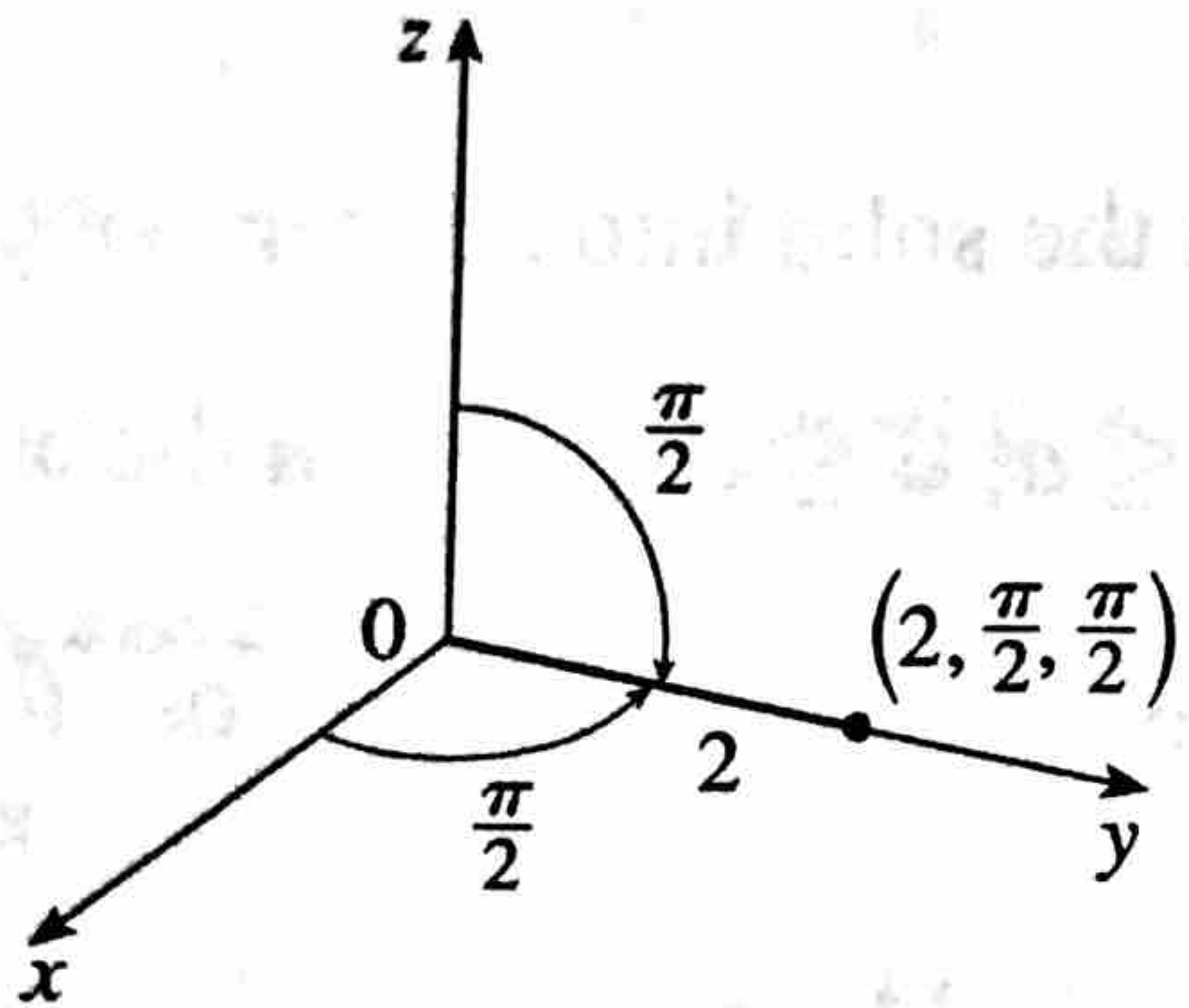
Thus the distance is $d = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|} = \frac{1}{\sqrt{6}} \sqrt{81 + 64 + 49} = \sqrt{\frac{194}{6}} = \sqrt{\frac{97}{3}}$.

53. (a) No. If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = 0$, so $\mathbf{a}$ is perpendicular to $\mathbf{b} - \mathbf{c}$, which can happen if $\mathbf{b} \neq \mathbf{c}$. For example, let $\mathbf{a} = \langle 1, 1, 1 \rangle$, $\mathbf{b} = \langle 1, 0, 0 \rangle$ and $\mathbf{c} = \langle 0, 1, 0 \rangle$.

(b) No. If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ then $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{0}$, which implies that $\mathbf{a}$ is parallel to $\mathbf{b} - \mathbf{c}$, which of course can happen if $\mathbf{b} \neq \mathbf{c}$.

(c) Yes. Since $\mathbf{a} \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{b}$, $\mathbf{a}$ is perpendicular to $\mathbf{b} - \mathbf{c}$, by part (a). From part (b), $\mathbf{a}$ is also parallel to $\mathbf{b} - \mathbf{c}$. Thus since $\mathbf{a} \neq \mathbf{0}$ but is both parallel and perpendicular to $\mathbf{b} - \mathbf{c}$, we have $\mathbf{b} - \mathbf{c} = \mathbf{0}$, so $\mathbf{b} = \mathbf{c}$.

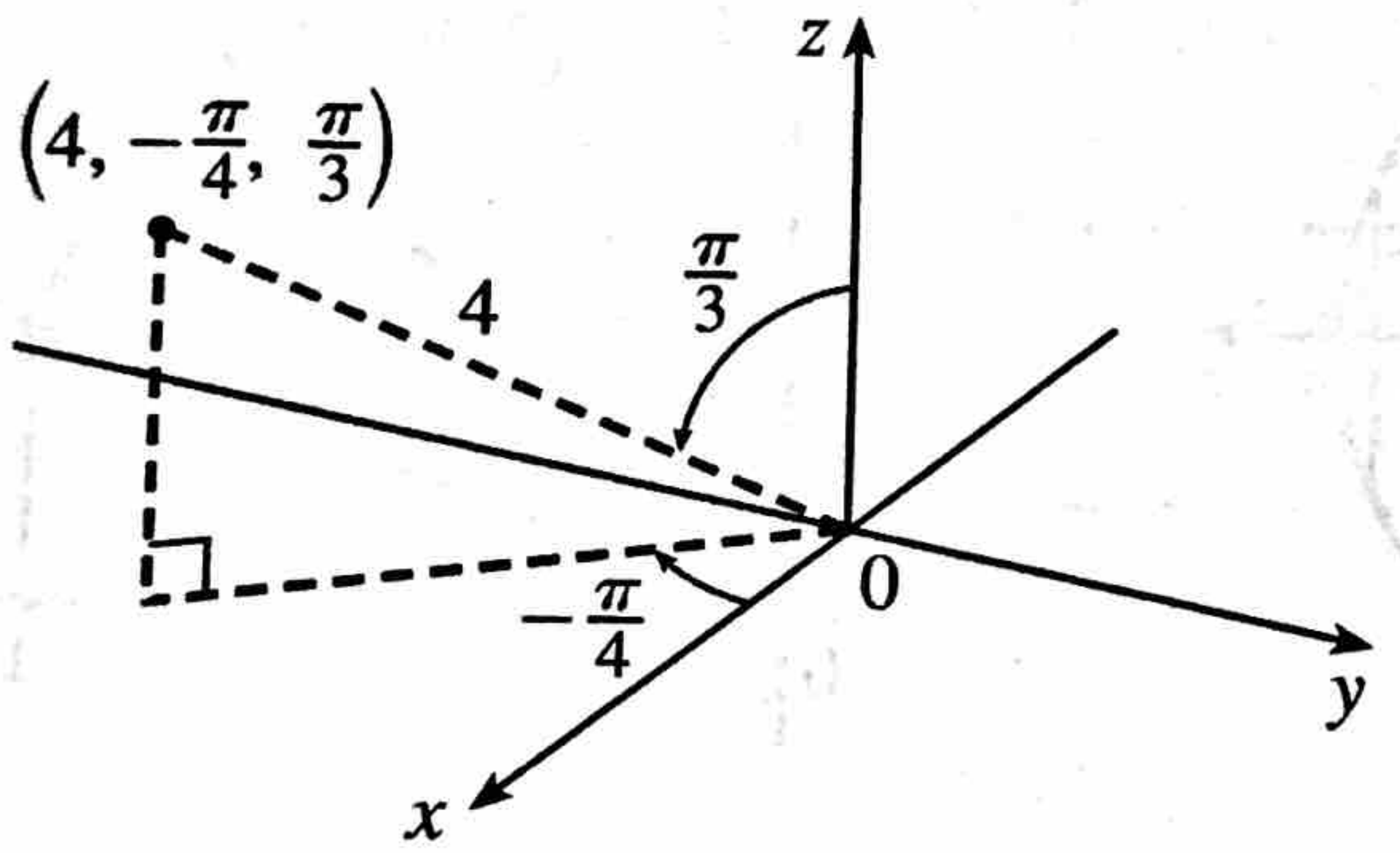
2. (a)



$x = 2\sin\frac{\pi}{2}\cos\frac{\pi}{2} = 2\cdot 1\cdot 0 = 0$, $y = 2\sin\frac{\pi}{2}\sin\frac{\pi}{2} = 2\cdot 1\cdot 1 = 2$,

$z = 2\cos\frac{\pi}{2} = 2\cdot 0 = 0$ so the point is $(0, 2, 0)$ in rectangular coordinates.

(b)



$x = 4\sin\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) = 4\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \sqrt{6}$,

$y = 4\sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) = 4\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{6}$,

$z = 4\cos\frac{\pi}{3} = 4\cdot\frac{1}{2} = 2$ so the point is $(\sqrt{6}, -\sqrt{6}, 2)$ in rectangular coordinates.

4. (a) $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{1 + 0 + 3} = 2$, $\cos\phi = \frac{z}{\rho} = \frac{\sqrt{3}}{2} \Rightarrow \phi = \frac{\pi}{6}$, and $\cos\theta = \frac{x}{\rho\sin\phi} = \frac{1}{2\sin(\pi/6)} = 1 \Rightarrow$

$\theta = 0$. Thus spherical coordinates are $\left(2, 0, \frac{\pi}{6}\right)$.

(b) $\rho = \sqrt{3 + 1 + 12} = 4$, $\cos\phi = \frac{z}{\rho} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \Rightarrow \phi = \frac{\pi}{6}$, and $\cos\theta = \frac{x}{\rho\sin\phi} = \frac{\sqrt{3}}{4\sin(\pi/6)} = \frac{\sqrt{3}}{2} \Rightarrow$

$\theta = \frac{11\pi}{6}$ [since $y < 0$]. Thus spherical coordinates are $\left(4, \frac{11\pi}{6}, \frac{\pi}{6}\right)$.

7. From Equations 1 we have $z = \rho \cos \phi$, so $\rho \cos \phi = 1 \Leftrightarrow z = 1$, and the surface is the horizontal plane $z = 1$.

8. $\rho = \cos\phi \quad \Rightarrow \quad \rho^2 = \rho\cos\phi \quad \Leftrightarrow \quad x^2 + y^2 + z^2 = z \quad \Leftrightarrow \quad x^2 + y^2 + z^2 - z + \frac{1}{4} = \frac{1}{4} \quad \Leftrightarrow \quad x^2 + y^2 + (z - \frac{1}{2})^2 = \frac{1}{4}$.

Therefore, the surface is a sphere of radius $\frac{1}{2}$ centered at $(0, 0, \frac{1}{2})$.